at one extremity, and limited by the radius produced through the other extremity.

37. The secant of an arc is the straight line joining the centre of the circle and the further extremity of the tangent drawn from the origin of the arc.

38. The sine, tangent, and secant of the complement of an arc are called the cosine, cotangent and cosecant of that arc.

Thus B C is the complement of the arc A B; B M D is the supplement of A B; angle B O C is the complement of A O B, and B O D is the supplement of A O B; B E is the sine of A B; A F is the tangent of A B; O F is the secant of A B; so B G is the sine of B C, or the cosine of A B; C H is the tangent of B C, or the cotangent of A B; and O H is the secant of B C, or the cosecant of A B. See figure 2.

39. The sine, tangent, and secant of an arc, are the sine, tangent, and secant of the angle measured by the arc.

Thus, the arc A B measures A O B and B E; A F, O F; the sine, tangent, and secant of the arc A B are also the sine, tangent, and secant of the angle A O B.

40. The sine, tangent, and secant of an angle, are the cosine, cotangent and cosecant of the complement of that angle.

Thus, B G or its equal O E the sine of angle B O C, is the cosine of A O B; C H and O H the tangent and secant of B O C, are the cotangent and cosecant of A O B.

41. The sine, tangent, and secant of an arc, are equal to the sine, tangent, and secant of its supplement.

Thus, B E, the sine of A B, is also the sine of B M D; A F, the tangent of A B, is equal to the tangent of A I L, which is equal to B M D.

Hence, when an angle is obtuse, its supplement must be used.

Propositions.

42. When the hypothenuse of a right-angled triangle is made radius, its sides become the sines of the opposite angles, or the cosines of the adjacent angles.

Thus, if A C be considered as radius, it is evident, by completing the figure (Art. 38), that B C is sine angle A or cosine angle C, and that A B is sine angle C or cosine angle A. See figure 3.

43. When the base is made radius, the perpendicular becomes the tangent of its opposite angle, and the hypothenuse the secant of the same; or the perpendicular becomes the cotangent of the adjacent angle, and the hypothenuse the cosecant of the same.

Thus, when A B is made radius, B C becomes tangent angle A or cotangent angle C, and A C becomes secant angle A or cosecant angle C. See figure 4.

44. When the perpendicular is made radius, the base is tangent of its opposite angle, and the hypothenuse secant of the same; or the base is cotangent of its adjacent angle, and the hypothenuse the cosecant of the same.

Thus, when B C is made radius, A B becomes tangent angle C or cotangent angle A, and A C becomes secant angle C or cosecant angle A. See figure 5.

Rules for Computation.

45. CASE I.—When a side and one of the oblique angles are given to find a side.

Rule.—Make any side radius: then
As the name of the given side
Is to the given side,
So is the name of the side required
To the side required.

46. CASE II.—When two sides are given to find an angle.

Rule.—Make one of the given sides radius: then
As the side made radius
Is to radius,
So is the other given side
To sine, tangent, or secant of the angle by it represented.

Note.—In working by the scale, let it be remembered that for sine, tangent, and secant, their numerical values to radius 60 must be used.

The examples in Trigonometry, Navigation, &c., are taken out of Norie's Navigation.

47. *Ex.*—Given the hypothenuse 370 miles, the angle A 56° 30′ and consequently the angle C 33° 30′ required the base A B and the perpendicular B C.

Making A C radius, A B will be sine angle C or cosine angle A, and B C will be sine angle A or cosine angle C. See figure 6.

Radius : Side A C 370 = Sine angle A 56° 30 : Side B C.
— — = Sine angle C 33° 30 : Side A B.

By the scale:

First radius is equal to 60, and sine 56° 30 is equal to 50. Then by Art. 14, set radius or 60 on index to 370 on side A, then opposite sine angle A 50 on index is found B C, 308.5 on A. And if the sine of angle C 33° 30 which is 33.3, be taken on index, by the one setting we find opposite it on A the side A B 204.2.

Or, set the index to 33° 30′ on quadrant, then opposite 370 on index is found on A the side B C 308.5, and on B the side A B 204.2.

48. Given the base A B 625 and angle A 48° 45, to find the hypothenuse A C and the perpendicular B C; *making A B radius, B C becomes tangent angle A or cotangent angle C, and A C becomes secant angle A or cosecant angle C.* See figure 7.

Then radius : A B 625 = Tangent angle A 48° 45′ : B C.
— — = Secant angle A 48° 45′ : A C.

By the scale:

Set 625 on index to radius 60 on A, then opposite tangent 48° 45′ or 68.4 on A is B C 713 nearly on index, and opposite secant 48° 45′ or 91 on A is found A C 948 nearly on index.

Or, set index to 48° 45′ on quadrant, and opposite 625 on A is 948 on index; and if the parallel from 948 on index be traced to B, it shows on B the perpendicular 713.

49. Given the hypothenuse A C 400, and the base B A 236; required the angles A and C, and the perpendicular B C.

Making the hypothenuse radius, B C becomes sine angle A or cosine angle C, and A B sine angle C or cosine angle A. See figure 8.

Then, A C 400 : Radius = A B 236 : Sine angle C.

By the scale:

Set radius 60 on index to 400 on A, then opposite 236 on A will be found sine angle C 35.4 on index: if the parallel of 35.4 on B be traced to quadrant, it will show on quadrant angle C 36° 9′, which, being subtracted from 90°, gives angle A 53° 51′.

Or, set 400 on index to 236 on side B, then the perpendicular from 400 on index to . ., shows on A the perpendicular B C 323, and the index cuts the quadrant in 36° 9′, which is the angle required.

Note.—The angle can be found more easily by making either of the sides about the right angle radius, when possible, as will be seen by next problem.

50. Given the base B A 35.5 and the perpendicular B C 41.6; required the angles A and C and the hypothenuse A C.

Making A B radius, B C will be the tangent of angle A or cotangent of angle C, and A C will be secant of angle A or cosecant of angle C. See figure 9.

Then A B 35.5 : Radius 60 = B C 41.6 : Tangent angle A.

By the scale:

Set 60 on index to 35.5 on A, then opposite 41.6 on A is found on index tangent angle A 70.2; then, if index be set to 70.2 on the line of tangents, it will cut the quadrant at 49° 31′, which is the number of degrees on angle A; and if the perpendicular from 35.5 on A be traced to index, it will show on index the hypothenuse A C 54.7, nearly.

Or, trace the perpendicular of 41.6 on side A 'till it will intersect the parallel of 35.5 on side B; set the index to the point of intersection: at this point will be found the hypothenuse 54.7 on index, and on the quadrant will be found the angle C 41° 29′, which, being subtracted from 90°, leaves angle A 49° 31′.

Examples for Exercise.

1. Given the hypothenuse 108 and the angle opposite the perpendicular 25° 36′; required the base and the perpendicular.

Ans.—The base is 97.4 and the perpendicular 46.66.

2. Given the base 96 and its opposite angle 71° 45′; required the perpendicular and the hypothenuse.

Ans.—The perpendicular is 31.66 and the hypothenuse 101.1.

3. Given the base 360 and the perpendicular 480; required the angles and the hypothenuse.

Ans.—The angles are 53° 8′ and 36° 52′, and the hypothenuse 600.

OBLIQUE-ANGLED TRIGONOMETRY.

51. CASE I.—When two angles and a side opposite to one of them are given.

Rule.—As the sine of the angle opposite to the given side is to the given side, so is the sine of the angle opposite to the required side, to the required side.

52. CASE II.—When two sides and an angle opposite to one of them are given.

Rule.—As the side opposite to the given angle is to the given angle, so is the side opposite to the required angle, to the required angle.

Note.—When two of the angles are known, the third is found by subtracting their sum from 180°.

Ex. 1.—Given angle A 38° 15′, and the angle B 105° 30′, and the side A B 53; required the sides A C and B C. See figure 10.

As sine angle 38° 15′	=	37.2 on B.
Is to its opposite side	"	53 on F.
So is sine angle 105° 30′	"	57.8 on B.
To its opposite side	"	82.5 on F.
And so is sine of sup. 36° 15°	"	35.5 on B.
To its opposite side	"	50.6 on F.

Set 53 taken on index to 37.1 on B; then opposite 57.8 on B is found on index A C 82.5, and opposite 35.5 on B is found on index side B C 50.6.

Ex. 2.—Given the side A B 336, the side B C 355, and the angle A 49° 26′; required the angles B and C and the side A C. See figure 11.

As side given 355	=	355 on B.
Is to sine of its opposite angle 49° 26′	"	45.5 on F.
So is the other given side	"	336 on B.
To sine of its opposite angle 45° 58′	"	43.1 on F.
And without a move so is		
Sine of supplement 84° 36′	"	59.8 on F.
To its opposite side	"	466 + on B.

Set 45.5 on index to 355 on side B, then opposite 336 on B is found sine angle C 43.1 on index: trace the parallel of 43.1 on B, till it intersect the quadrant, and at the point of intersection is found on the quadrant 45° 58′, the angle at C.

If the angles A and C be now added, and the sum subtracted from 180°, the remainder is angle B 84° 36′: then, by the first case, A C can be found.

53. CASE III.—When two sides and the angle contained between them are given.

Rule.—As the sum of the two given sides is to their difference, so is tangent of half the sum of the unknown angles to the tangent of half their difference. This half difference, added to half their sum, gives the greater angle, and subtracted, leaves the less. The angles being thus all known, the remaining side is found by Rule to Case I.

Ex.—Given the side A B 85, the side A C 47, and the angle A 52° 40′; required the angles C and B and the side B C. See figure 12.

$$\begin{array}{lcl} & & 180^\circ \\ \text{Angle A} & = & 52^\circ\ 40 \\ \hline \text{B} + \text{C} & = & 127^\circ\ 20' \\ \hline \tfrac{1}{2}\,(\text{B} + \text{C} & = & 63^\circ\ 40' \end{array}$$

(A B + A C) 132 : (A B − A C) 38 = Tang. ½ (C + B) 63° 40′ : Tang. ½ (C − B).

Here, we must use the semi-tangent, found (Art. 23) on the perpendicular of 30 on A, to be 60.6; and on trial the operator will find it necessary to employ 66 and 19 in the first two terms of the proportion, instead of 132 and 38.