For let the curve lines BK′ and BK be supposed each to move on the point B as a center equally towards each other, they must meet on, and coincide with the curve line BGD′—because the arcs KD′ and K′D′, are equal by construction, and the curvilinear triangles ts′s and sr′r, and the curvilinear triangles tu′u and uv′v, will coincide with the curvilinear triangles *de′e* and *ef′f*—also the arcs of variation t′s″ and tu″, will each coincide with the arc de″—and the arcs sr″and uv″ each coincide with the arc *ef″*—hence their ultimate ratios are equal (Lem. 8) ; consequently the variation t′s″ is to the variation sr″,—as the variation tu″ is to the variation uv″,—as the variation de″ is to the variation *ef″*; therefore de″ and *ef″* must be the arcs of variation, on the common curve line, or arc dk, of the point d towards *e*, and of the point *e* towards *f*.

LEMMA 10.

FIG. 7. From the point B as a center, with the distances Bk′ and B*i*′, describe the arcs k′k″ and *i′i″*,—the arc kk″, shall be the variation of k towards *i*, and the arc *ii″* shall be the variation of *i* towards h.

For from the point A, with the distances Ao′ and Aq′, describe the arcs o*x*′ and qq′; and from the point B as a center, with the distances Bq′ and B*x*′, describe the arcs q′q″ and *x′x″* ; Also from the point C as a center, with the distances CK and Co, describe the arcs Ky′ and op,′ and from the point B, with the distances Bp′ and By′, describe the arcs p′p″ and y′y″.

Now let the curve line BK and BK′, each move upon the point B as a center, towards the curve line BD′, they must each coincide with BD′, (Lem. 9,)—hence the triangles qq′o′ and o′*x*′K′ and the triangles qp′o, and oy′K, will coincide, and be similar and equal to the triangles h*i*′*i*, and *i*k′k ; consequently their ultimate ratios are equal, and the variation Ky″, is to the variation op″, as the variation K′x″, is to the variation o′q″, as the variation kk″, is to the variation *ii″*, (Lem. 9,)—therefore kk″ must be the variation of the point k towards *i*, and *ii″*, the variation of the point *i* towards h.

THE SOLUTIONS.

Case First.—*When the arc Bdef ... hikD′, is a curve line, through intersections described through points of equal distances on the arcs BD and BC*, (*Lem.* 1, *Fig.* 1.)

SOLUTION FIRST.

PROPOSITION 1.

THEOREM.

FIG. 7. From the point B with the distances Bk and B*i*, and from the point D′, with the distances D′*e″* and D′*f″*, describe the intersections a and b, and through the points a and b, draw the straight line abn meeting the curve line BGD′ in the point n.—The straight line abn, shall be that upon which, by variation of the points d and *e*, to the points *e″* and *f″* the curve line abc ... n (fig. 6), shall coincide with its chord an.

For as the curves V′p′q ... n″, and Vpq ... n′, are each upon the opposite side of theirchords, to the curve a′b′c′ ... n on the arc BD (fig. 5)—it is evident that intersections described through the points of variation n″ and v″, and s″ and r″, (fig. 7,) must give the curve lines on the same side of their chord as Vpq ... n′ and Vpq ... n″—and therefore be also on the opposite side to the curve line a′b′c′ ... n (fig. 5)—because the variations tu″ and uv″ are proportional to tu and uv—and ts″ and sr″ are proportional to ts and , and therefore as the curve line described by intersections through the points n″ and v″ shall vary, the arc BK′ will move towards BC on the center B, and that described through the points s″ and r″, the arc BK, must move towards BD on the center B, (Lem. 10)—so that the curves of intersections described through the points of variations n″ and v″, and s″ and r″, will come to be, each in one straight line with their chords; Now it is evident from their ultimate ratios being equal, that the distances K′n″ will become equal to Ks″, and Kv″ equal to Kr″, and *BK″* equal to *BK*, and *Bo* equal to *Bo′*; but this can only be possible on the curve line BD′, on which is the intercepted arc dk common to both of the series of arcs 1K and 1″K′, and upon dk are the common variations de″ and *ef″*, (Lem. 9,) ; consequently it is through the points *e″* and *f″* and h and *i*, in this case, that the intersections described from the points B and D′, forming the curve line ab ... n will only coincide with the chord an.