

point at or near the centre of the span, where the slope is zero, and the support. If this statical moment is expressed in terms of the maximum ordinate of the  $M/EI$  curve and the whole span, an expression is obtained of the form of equation (3), in which the coefficient  $k$  will equal  $k_1/k_2$ .

It may be of interest here to point out that  $M/EI$  equals the rate of change of the slope of the elastic curve, or the change of slope per unit distance. This may be seen by dividing equation (1) by  $dS$ , which gives  $d\theta/dS = M/EI$ .

The application of this principle to the deflection of reinforced-concrete beams consists in finding a convenient expression for the value of  $M/EI$ .

Let  $e_o$  and  $c_o$  be the unit deformation in the extreme fiber and the distance from the extreme fiber to the neutral axis, respectively, for the concrete, and let  $e_s$  and  $c_s$  be the same quantities for the steel. In a homogeneous beam, as above indicated,  $e/c = M/EI = d\theta$ , and is the same for both extreme fibers. In this case, since the

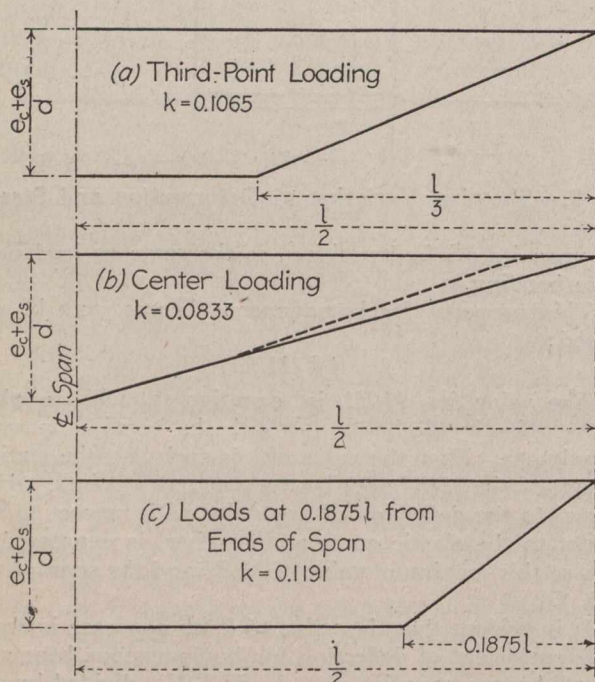


Fig. 3.—Showing Variation in Values of  $(Ec + Es)/D$ .

values of  $EI$  are constant throughout the length of the beam,  $e/c$  varies directly as  $M$  throughout the span.

In a reinforced-concrete beam (see Fig. 2),

$$\frac{M}{EI} = d\theta = \frac{e_o}{c_o} = \frac{e_s}{c_s} = \frac{e_o + e_s}{d}$$

Substituting this value of  $M/EI$  in equation (3), the following expression for the deflection of a reinforced-concrete beam is obtained:

$$f = k \frac{l^2}{d} (e_o + e_s)$$

This is based on the usual assumption that a plane before bending remains a plane after bending, which seems justifiable from the evidence of reliable tests herein referred to.

From the preceding analysis it is evident that the deformations of the extreme fibers are the only determining factors in the deflection, except the span, depth of beam, and load-distribution. It is also evident that the distribution of the stresses in the steel and concrete over the section has no influence on the deflection, except in

so far as the stresses influence the deformations of the extreme fibers.

The influence of tension in the concrete might well be discussed here. From the principles of equilibrium it is known (see Fig. 2) that the effect of the tension in the concrete at low stresses is to reduce to some extent the compressive stresses in the concrete and the tensile stresses in the steel. This means that a stiffer beam might be expected in the earlier stages of the loading, as the steeper slope of deflection and deformation curves show.

For the reason, therefore, that tension exists in the concrete and that usually near the end of the beam the roads are bent up, the value of the  $(e_o + e_s)/d$  curve, from which the deflection is obtained, will not have the same variation as the values of  $M$ .

Near the supports of a simple beam and at all points where the bending moment is small, we would expect considerably smaller values of  $(e_o + e_s)/d$  than the value of  $M$  at such a point, relative to the value of  $M$  at the point of maximum moment would indicate. The values of  $(e_o + e_s)/d$  would probably follow the dotted lines indicated in Fig. 3b, because of the tension in the concrete. A glance at Fig. 3b will show, and computations will prove that a small decrease in values of  $(e_o + e_s)/d (= d\theta)$  near the point about which moments are taken (the support, in this case), changes the value of this moment only slightly.

Values of  $k$  for maximum deflections under several conditions of loading are here given:

Beam with uniform load:

Ends freely supported,  $k = 5/48$  or 0.1041;

Ends fixed,  $k = 1/32$  or 0.0313.

Beam loaded at the third points:

Ends freely supported,  $k = 23/216$  or 0.1065;

Ends fixed,  $k = 5/144$  or 0.0347.

Beam loaded at the middle:

Ends freely supported,  $k = 1/12$  or 0.0833;

Ends fixed,  $k = 1/24$  or 0.0416.

## DOMINION GOVERNMENT BUYS INTERNATIONAL RAILWAY.

On August 1st the International Railway became a part of the Intercolonial Railway and will be operated by the Dominion Government in future. Mr. Evan Price, superintendent of the Canada Eastern Division of the Intercolonial Railway, will superintend the operation of the new portion, which is 112 mi. in length.

## OGDEN POINT BREAKWATER, VICTORIA, B.C.

The site of the proposed breakwater at Ogden Point is now the scene of considerable activity, the contractors having started work last week. Rip-rap is being hauled by towboats and scows from the Albert Head Quarries. The facilities for the transportation of this material are adequate for conveying 50,000 tons per month. Pile-driving to indicate the line of the breakwater has been completed, the extreme point being about 1,750 ft. from shore. The shore work in connection with the breakwater is almost complete, the site having been levelled off and the fillings completed. These fillings have greatly enlarged the land surface at the disposal of the government for the piers in connection with the breakwater.

It is expected that the work will be nearing completion in about eighteen months. The breakwater is of concrete construction, faced with large granite blocks.