i.e.,

$$\frac{A}{(b-c)(1+a^2)}=\frac{B}{(c-a)(1+b^2)}=\frac{C}{(a-b)(1+c^2)}$$

$$\therefore \frac{\frac{A}{1+a^2}}{b-c}=\frac{\frac{B}{1+b^2}}{c-a}=\frac{\frac{C}{1+c^2}}{a-b}$$

$$=\frac{\frac{A}{1+a^2}+\frac{B}{1+b^2}+\frac{C}{1+c^2}}{b-c+c-a+a-b}=0,$$

$$\therefore \frac{A}{1+a^2}+\frac{B}{1+b^2}+\frac{C}{1+c^2}=0.$$

Again,

$$\frac{A}{a(b-c)(a+\frac{1}{a})}=\frac{B}{b(c-a)(b+\frac{1}{b})}=\frac{C}{c(a-b)(c+\frac{1}{c})}$$

$$\therefore \frac{\frac{A}{a+\frac{1}{a}}}{a(b-c)}=\ldots=\frac{\frac{A}{a+\frac{1}{a}}+\frac{B}{b+\frac{1}{b}}+\frac{C}{c+\frac{1}{c}}}{ab-ac+bc-ab+ac-bc}=0.$$

$$\therefore \frac{A}{a+\frac{1}{a}}+\frac{B}{b+\frac{1}{b}}+\frac{C}{c+\frac{1}{c}}=0.$$

123. If $\frac{xh}{a^2}=\frac{yk}{b^2}=\frac{zl}{c^2}$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$,

prove that $\left(\frac{x}{h}+\frac{y}{k}+\frac{z}{l}\right)^2=\frac{a^2}{h^2}+\frac{b^2}{k^2}+\frac{c^2}{l^2}$

$$\frac{xh}{a_2}=\frac{\frac{x}{h}}{\frac{a^2}{h^2}},\quad \frac{yk}{b^2}=\frac{\frac{y}{k}}{\frac{h^2}{k^2}},\quad \frac{zl}{c^2}=\frac{\frac{z}{l}}{\frac{c^2}{l^2}}$$

$$(1)\quad \therefore \frac{xh}{a^2}=\frac{yk}{b^2}=\frac{zl}{c^2}=\frac{\frac{x}{h}+\frac{y}{k}+\frac{z}{l}}{\frac{a^2}{h^2}+\frac{b^2}{k^2}+\frac{c^2}{l^2}},$$

$$(2)\text{ and } \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

$\therefore$ Dividing the terms in (2) by the corresponding terms of (1),

$$\frac{x}{h}+\frac{y}{k}+\frac{z}{l}=\frac{\frac{a^2}{h^2}+\frac{b^2}{k^2}+\frac{c^2}{l^2}}{\frac{x}{h}+\frac{y}{k}+\frac{z}{l}},$$

$$\therefore \left(\frac{x}{h}+\frac{y}{k}+\frac{z}{l}\right)^2=\frac{a^2}{h_2}+\frac{b^2}{k^2}+\frac{c^2}{l_2}.$$

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124. What is the length of an edge of the largest cube that can be cut from a sphere 40 inches in diameter?

125. Any number is divisible by 11, if, counting from the units place, the sum of the digits in the odd places is equal to the sum of the digits in the even places, or if the difference between these sums is divisible by 11. Give mathematical proof of this.

126 A circular grass plot whose area is one-quarter of an acre, has erected at its centre a pole 12 feet high, and of the uniform diameter of one foot. Attached to the top of this pole is one end of a cord, the length of which is just sufficient to allow the other end to touch the edge of the plot. The cord is then wound spirally on the post so as to make one complete revolution in every foot of its descent. When it has been thus wound from the top to the bottom of the post, what is the area of the circle, in square yards, of which the unwound part of the cord is radius.

127. If x be any odd number greater than unity, shew that (x^5-x) is divisible by 24; also that $(x^2+3)(x^2+7)$ is divisible by 32.

128. The sum of the squares of three consecutive numbers, when increased by unity, is divisible by 12 but never by 24.

129. If $x=(p+q)^2$, find value of
$$(p^2+q^2)x-2pqx-(q^4+p^4)$$

130. Shew that
$$(x-y)^3+(y-z)^3+(z-x)^3$$
$$=3(x-y)^3+3(y-z)^3+3(z-x)^3$$
$$-6(x-y)(y-z)(z-x).$$

131. Shew that
$$abc>(a+b-c)(a+c-b)(b+c-a).$$

If one relation exists between the values of a, b and c, this proposition is not true; point out that relation.

132. Two spheres, whose radii are x and y, touch each other internally. Find the distance of the centre of gravity of the solid contained between the two surfaces, from the point of contact.

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