(3).....$\log (a^n) = n \log a.$

Let $x = \log a$, so that $10^x = a.$

Then,

$$a^n = (10^x)^n = 10^{nx}$$

so that nx is the logarithm of a^n

or $$\log (a^n) = n \log a.$$

(4) .....$\log ({}^n\sqrt{a}) = \frac{1}{n} \log a.$

Let $x = \log a$, so that $10^x = a.$

Then $${}^n\sqrt{a} = a^{\frac{1}{n}} = (10^x)^{\frac{1}{n}} = 10^{\frac{x}{n}},$$

so that $\frac{x}{n}$ is the logarithm of ${}^n\sqrt{a}$,

or $$\log ({}^n\sqrt{a}) = \frac{1}{n} \log a.$$

6. Any of these operations may be combined: thus

$$\log (abcd) = \log a + \log b + \log c + \log d;$$

$$\log \left(\frac{a}{b\,c}\right) = \log a - \log b - \log c;$$

$$\log \frac{a}{c^2} \cdot \frac{\sqrt{b}}{\sqrt[3]{d}} = \log a + \tfrac{1}{2} \log b - 2 \log c - \tfrac{1}{3} \log d.$$

The mantissa independent of the place of the decimal point in the number.

7. *The mantissa of the logarithm is the same for all numbers which differ only in the position of the decimal point.*

For any change in the position of the decimal point in a number is effected by a continued multiplication or division by ten; and since $\log 10 = 1$, each such multiplication or division alters the characteristic of the logarithm only by the addition or subtraction of 1, thus leaving the mantissa unchanged.