

Given expression =  $\frac{(x^2 - y^2)s^2 + \dots}{(y-x)(x-x)(x-y)}$   
 $= yz + zx + xy.$

5. Extract the square root of

(1)  $2 \left( 1 - \frac{b^2 + c^2 - a^2}{2bc} \right)$   
 $\left( 1 - \frac{c^2 + a^2 - b^2}{2ca} \right) \left( 1 - \frac{a^2 + b^2 - c^2}{2ab} \right).$

(2)  $x^4 + x^3 + \frac{29}{4}x^2 + \frac{7}{2}x + \frac{49}{4}.$

(1) Given expression

$$= \frac{(a^2 - b - c^2)(b^2 - c - a^2)(c^2 - a - b^2)}{4a^2b^2c^2}$$

whose sq. root is

$$\frac{(b+c-a)(c+a-b)(a+b-c)}{2abc}$$

(2)  $x^2 + \frac{x}{2} + \frac{7}{2}.$

6. Find the value of  $x$  in

$$(x+a)(b-c) + (x+b)(c-a) + (x+c)(a-b) = 0.$$

Explain result.

$$x = \frac{a(b-c) + b(c-a) + c(a-b)}{b-c+c-a+a-b} = 0.$$

The value of  $x$  is indeterminate.

7. Find an expression for  $k$  in terms of  $a, b, c$ , that may make

$$\frac{b^2 - c^2}{k-a} + \frac{c^2 - a^2}{k-b} + \frac{a^2 - b^2}{k-c}, \text{ vanish.}$$

Value of  $k$  is obviously  $a+b+c$ , for with this value, given expression

$$= b-c+c-a+a-b=0.$$

8. If

for every \$3 of income  $A$  has,  $B$  has \$2; for every \$12  $A$  spends,  $B$  spends \$1; and for every \$4  $A$  saves,  $B$  saves \$5; find the proportion of his income that  $A$  saves.

Let  $3x, 12y, 4z$  be  $A$ 's income, what he spends, and what he saves respectively, then  $2x, y, 5z$  are corresponding amounts for  $B$ ;

$$\therefore \left. \begin{array}{l} 3x = 12y + 4z \\ \text{and } 2x = y + 5z \end{array} \right\} 4z = \frac{3x}{2},$$

$A$  saves half his income.

9. Solve the equations

(1)  $\frac{x+1}{5} + x(x-1) = (x-1)^2.$

(2)  $\frac{1}{x-a} - \frac{1}{x-2a} = \frac{1}{x-3a} - \frac{1}{x-4a}.$

(3)  $\frac{2x^3 + 2x^2 + 3x + 1}{x^2 + x + 1} = \frac{x^2 - x + 1}{x-1} + \frac{x^4 - x + 1}{x^3 - 1}.$

(4)  $\left. \begin{array}{l} x^2 + xy + y^2 = 25 \\ x + xy + y^2 = 31 \end{array} \right\}.$

(1)  $x = \frac{2}{3}$  } On simplifying these equations, powers of  $x$ , higher than the first, vanish.

(2)  $x = \frac{5a}{2}$

(3)  $x = -3$

(4) Adding  $(x+y)^2 + (x+y) = 56, x+y = 7 \text{ or } -8,$

$$\left. \begin{array}{l} x = 3 \text{ or } -\frac{11}{3} \\ y = 4 \text{ or } -\frac{13}{3} \end{array} \right\}$$

EUCLID.

1. Shew clearly that in Book I. Euclid proves that if the three sides of a triangle be given, or two sides and the contained angle, then the triangle is determinate. (The proofs of the propositions in which this is made out are not required.)

Is there any other case in which Euclid shews that if certain parts be given the triangle is determinate?

2. If two parallel lines be also equal, the lines joining their ends are either parallel and equal or else they bisect one another.

State converses of these propositions, and prove one of such converses.

3. If a parallelogram be on the same base with a triangle, and both have the same altitude, the former is double the latter.

4. Shew that the square on the hypotenuse of a rightangled triangle is equal to the sum of the squares on the sides.

5.  $ABCD$  is a quadrilateral having  $AD$  parallel to  $BC$ ; shew that if  $E$  be the bisection of  $AB$ , the triangle  $ECD$  is half the quadrilateral.

Shew also that if  $F$  be the bisection of  $AD$ , and  $FBC$  be half the quadrilateral, then the quadrilateral is a parallelogram.