

will not generally be constant, for instance, the instantaneous current due to the filling up of a condenser diminishes in strength as the potential of the condenser increases. Let, therefore, C be the current at any time t , then the moment of the deflecting couple is $bCML$, where b is a constant depending on the galvanometer, M the magnetic intensity of each pole of the needle and l the distance between the poles. Hence the equation of motion is

$$mk^2 \cdot d\omega = bCML \cdot dt$$

But C can be considered constant for the short interval of time dt , so that $C \cdot dt$ is the quantity of electricity, dQ , transmitted during the interval dt . Hence

$$mk^2 \cdot d\omega = bML \cdot dQ$$

Integrating and remembering that $Q=0$ when $\omega=0$, and writing Ω for ω

$$mk^2 \Omega = bMLQ$$

But the work done on the needle is $\frac{mk^2 \Omega^2}{2}$, hence

$$Q^2 \propto \text{work done on needle.}$$

To show that $\sin^2 \frac{\delta}{2}$ is a measure of the work done by an instantaneous current on the needle of a galvanometer.

The effect of an instantaneous current is, as seen above, to impress an initial angular velocity on the needle. During the motion, the forces acting on the needle are: the magnetic attraction of the Earth, (forming a couple,) the resistance of the air and the friction of the pivot. The last two forces will, however, be neglected and their effect allowed for by a subsequent correction.*

Let ω be the angular velocity at any time t , and θ the angle described from the position of equilibrium; the remaining letters having the same signification as in the previous paragraph. Then the equation of motion, obtained by taking moments about the pivot, is

$$mk^2 \frac{d^2 \theta}{dt^2} = -HML \sin \theta$$

Multiplying both sides of this equation by $2 \frac{d\theta}{dt}$, and integrating

$$mk^2 \left\{ \frac{d\theta}{dt} \right\}^2 = 2HML \cos \theta + \text{constant.}$$

But, $\frac{d\theta}{dt} = 0$, when $\theta = \delta$, the angle of deflection. Hence

$$mk^2 \left\{ \frac{d\theta}{dt} \right\}^2 = 2HML (\cos \theta - \cos \delta)$$

*See § 180.