If the column is very short the material will be crushed and there will be an even pressure over both channels, as shown by Fig. 2, and equal to S_0 per square inch.

Fig. 2.

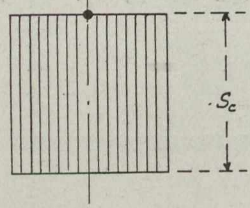
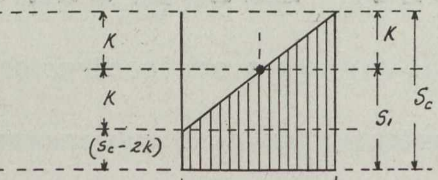


Fig. 3.



If the column is long, it will fail by bending and there will be tension on one side and compression on the other, or, by referring to Fig. 3, it will be seen that there is a difference of stress between the two channels equal to $2k$.

Or the total stress in channel marked C.R. would be $2k\frac{A}{2}$, due to bending.

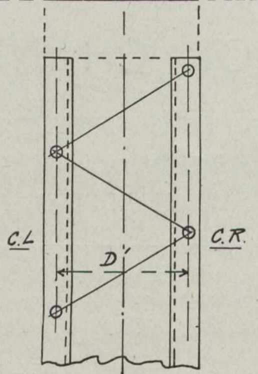


Fig. 4.

Let S_0 = 16,000 lbs. = allowable pressure per square inch when column is very short, and S_1 = allowable average pressure per square inch for long columns.

Then, $(S_0 - S_1)$ = stress per square inch due to bending of column at centre = k, and $(S_0 - S_1)A$ = total stress in column at centre due to bending of column. [Equation 1.]

A = area of column.

Referring to Merriman's Mechanics of Materials, 1894 edition, page 132, he gives what he considers the most accurate column formula for

$$\frac{P}{A} = S_1, \text{ namely,}$$

$$\frac{P}{A} = \frac{S_0}{1 + \frac{S_0}{m\pi^2 E}\frac{l^2}{r^2}} \quad \text{[Equation 2.]}$$

Substituting usual values in Equation 2,

$$S_1 = \frac{16{,}000}{1 + \frac{1}{12{,}000}\frac{l^2}{r^2}}. \quad \text{[Equation 3.]}$$

Referring to Fig. 3 and Fig. 4 it will be noticed that the right-hand channel (marked C.R.) has a stress per square inch = S_0, while channel marked C.L. has a stress per square inch = $S_0 - 2k$, making a difference in stress per square inch between the two channels of $2k$ per square inch.

But $k = (S_0 - S_1)$. [Equation 4.]

Therefore, $2k = 2(S_0 - S_1)$. [Equation 5.]

Then the total stress in channel (marked C.R.) due to bending is $= 2k\frac{A}{2} = 2(S_0 - S_1)\frac{A}{2}$. [Equation 6.]

Now referring to Fig. 1:—

When a long column is loaded it will bend as stated before, and the curve it will take will be a sinusoid whose equation is $y = \Delta \sin \pi \frac{x}{l}$. [Equation 7.] (Hinged ends only. If column is not hinged top and bottom, then the general equation for the flexure of the column will become $y = \Delta \sin n\pi x/l$. If one end is fixed and the other end is round, then $n = 2$. If both ends are fixed, then $n = 3$. The above equations would have to be used in place of Equation 7.)

(See Merriman's Mechanics of Materials, 1894 edition, page 115.)

Let M = bending moment at centre of column, then $M = P\Delta$.

Total stress in one channel due to

$$M = \frac{M}{D'}, \text{ and } \frac{M}{D'} = \frac{P\Delta}{D'}. \quad \text{[Equation 8.]}$$

But it was found in Equation 6 that the total stress in one channel was equal to $2(S_0 - S_1)\frac{A}{2}$.

$$\text{Therefore } \frac{P\Delta}{D'} = 2(S_0 - S_1)\frac{A}{2}. \quad \text{[Equation 9.]}$$

$$\text{Or, } \Delta = D'\frac{(S_0 - S_1)A}{P}. \quad \text{[Equation 10.]}$$

m = bending moment at any point x.

$= Py$. Substitute for y, (see Equation 7),

$$= P\Delta \sin \pi \frac{x}{l}. \quad \text{[Equation 11.]}$$

Substitute for Δ, (see Equation 10),

$$m = \frac{PD'(S_0 - S_1)A}{P} \sin \pi \frac{x}{l}.$$

$$\text{Or, } m = D'(S_0 - S_1)A \sin \pi \frac{x}{l}. \quad \text{[Equation 12.]}$$

Now, stress in channel at any point $x = \frac{m}{D'} = p$.

$$\text{Therefore, } \frac{m}{D'} = \frac{D'(S_0 - S_1)A}{D'} \sin \pi \frac{x}{l}.$$

$$\text{Therefore, } \frac{m}{D'} = (S_0 - S_1)A \sin \pi \frac{x}{l} = p. \quad \text{[Equation 13.]}$$

Referring to Fig. 5:—

The stress in end lattice bar $bc = p \sec \phi$.

Therefore, stress in end lattice bar

$$bc = (S_0 - S_1)A \sin \pi \frac{x}{l} \sec \phi. \quad \text{[Equation 14.]}$$

But there are two lattice bars; therefore,

$$bc = \frac{(S_0 - S_1)A \sin \pi \frac{x}{l} \sec \phi}{2}. \quad \text{[Equation 15.]}$$

But angle ϕ is usually 60° and the secant of 60° is 2; therefore, stress in each lattice bar

$$bc = (S_0 - S_1)A \sin \pi \frac{x}{l}. \quad \text{[Equation 16.]}$$

If Equation 16 is solved, it will give the stress in each end lattice bar when ϕ is 60°, and as the end bar

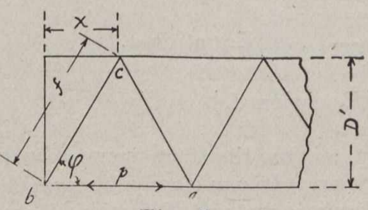


Fig. 5.

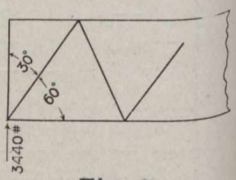


Fig. 6.

takes a maximum shear, all that it is necessary to do is to design the end bar and make the remainder of the bars the same size.

To find the stress in the lattice bars for any other angle than ϕ = 60°, use Equation 15, which is a general equation.