and it then follows again from the theory of differential equations of the second order that $y = y_1 + y_2$, that is,

$$y_1 = R.e^{-t/2T_0} \sin(\beta + t/T_1) \quad \text{general integral} \quad (73)$$

$$y_2 = b \sin(\psi + \frac{t}{\mathrm{T}}) \quad \text{particular integral} \quad (73)$$

The latter may also be written

$$y_2 = b.\sin\psi\cos\frac{t}{\mathrm{T}} + b\cos\psi\sin\frac{t}{\mathrm{T}} \quad (74)$$

The integration constants R and β may be obtained from the initial phase and the constants b and ψ by forming the equations

$$\frac{dy_2}{dt} = \frac{b}{\mathrm{T}}\cos(\psi + \frac{t}{\mathrm{T}}) = \frac{b}{\mathrm{T}}\cos\psi\cos\frac{t}{\mathrm{T}} - \frac{b}{\mathrm{T}}\sin\psi\sin\frac{t}{\mathrm{T}}$$

$$\frac{d^2y_2}{dt^2} = -\frac{b}{\mathrm{T}^2}\sin(\psi + \frac{t}{\mathrm{T}}) = -\frac{b}{\mathrm{T}^2}\sin\psi\cos\frac{t}{\mathrm{T}} - \frac{b}{\mathrm{T}^2}\cos\psi\sin\frac{t}{\mathrm{T}}$$

We then introduce these values in the differential equation and combine the members containing cos t/T and sin t/T

$$[-\frac{b}{\mathrm{T}^2}\sin\psi + \frac{b}{\mathrm{T}T_0}\cos\psi + \frac{b}{T^2}\sin\psi + \epsilon.f.c_1\sqrt{\frac{1}{T_0^2} + \frac{1}{\mathrm{T}^2}}.\sin\phi]\cos\frac{t}{\mathrm{T}} +$$

$$[-\frac{b}{\mathrm{T}^2}\cos\psi - \frac{b}{\mathrm{T}T_0}\sin\psi + \frac{b}{T^2}\cos\psi + \epsilon.f.c_1\sqrt{\frac{1}{T_0^2} + \frac{1}{\mathrm{T}^2}}.\cos\phi]\sin\frac{t}{\mathrm{T}} = 0$$

and as these equations hold good for all values of t, the terms in both parentheses must become 0. One thus obtains two equations with the unknowns b and ψ and considering that $tg\phi = \frac{T_0}{\mathrm{T}}$ and $h_1 = \frac{c_1 T^2}{T_0}$, it follows

$$b = -\epsilon.f.h_1 \frac{\sqrt{1 + \frac{T_0^2}{\mathrm{T}^2}[\frac{T^2}{\mathrm{T}^2} + \frac{T^2}{T_0^2} - 1]^2}}{1 + \frac{T^2}{\mathrm{T}^2}[\frac{T^2}{\mathrm{T}^2} + \frac{T^2}{T_0^2} - 2]} \quad (75)$$

$$tg\psi = -\frac{T_0}{\mathrm{T}}.[\frac{T^2}{\mathrm{T}^2} + \frac{T^2}{T_0^2} - 1]$$

For the determination of the integration constants R and β, it must be considered that for

$$t = 0; \quad z = -\epsilon h_1; \quad y = 0; \quad s = 0.$$

We obtain for this case the equations

$$R.\sin\beta = -b.\sin\psi$$

$$R.\cos\beta = -b[\frac{T_1}{\mathrm{T}}\cos\psi + \frac{T_1}{2T_0}\sin\psi] \quad (76)$$

During the variation of the outflow the following equations are effective:

$$z = \epsilon.h_1 + Re^{-\frac{t}{2T_0}}\sin(\beta + \frac{t}{T_1}) + b\sin(\psi + \frac{t}{\mathrm{T}})$$

$$s = \frac{R}{T}e^{-\frac{t}{2T_0}}\sin(\gamma - \beta - \frac{t}{T_1}) + \frac{b}{\mathrm{T}}\cos(\psi + \frac{t}{\mathrm{T}})$$

with $tg\gamma = \frac{2T_0}{T_1}$

ψ amounts generally to about $\frac{3\pi}{2}$ which relation simplifies the computation of R and β.

The variation ceases (as we assumed) after the time $t = \pi.\mathrm{T}$. The values which correspond to the elevation of the water surface and velocity at that time are obtained by the formulæ:

$$z_{\mathrm{T}} = -\epsilon h_1 + Re^{-\frac{\mathrm{T}}{2T_0}}\sin(\beta + \pi\frac{\mathrm{T}}{T_1}) - b.\sin\psi$$

$$s_{\mathrm{T}} = \frac{R}{T}e^{-\frac{\mathrm{T}}{2T_0}}\sin(\gamma - \beta - \pi\frac{\mathrm{T}}{T_1}) - \frac{b}{\mathrm{T}}\cos\psi \quad (77)$$

For further investigation, the formulæ of case (A) may be applied. In order to simplify matters, in the determination of the further movement, we measure the time from the instant of the beginning of the constant outflow and logically we must use the limiting values of the preceding phase for the determination of the integration constants R_1 and β_1.

If we do not hinder the variation of the outflow but maintain the law $\epsilon Q_1(1 + f\sin\frac{t}{\mathrm{T}})$, we see that the movement of the water surface in the surge tank takes the form of a forced oscillation; where the influence of the first member decreases with the increase of t and this the quicker the larger the value of $\frac{T_1}{2T_0}$ becomes in the member

$$e^{-\frac{t}{2T_0}} = e^{-\frac{T_1}{2T_0}.\frac{t}{T_1}}$$

The movement of the level of the water surface is merely that of a harmonic oscillation. In such cases it is well known that the phenomenon of resonance may occur, if the period of the actuating influence has the same duration as the swinging bodies' own period, that is to say, if, in the case mentioned $\mathrm{T} = T$. The value of the amplitude of the forced oscillation is then

$$b = -\epsilon fh_1\frac{\sqrt{1 + \frac{T^2}{T_0^2}}}{T^2/T_0^2} = -\epsilon fh_1\sqrt{\frac{T_0^2}{T^2} + 1}.\frac{T_0}{T} \quad (78)$$