the introduction of intermediate propositions essentially inadmissible. To be beguiled by the fallacy of *rigorous* reasoning from *erroneous* data, and illogical reasoning from true data, seems to be one of the most common of our intellectual failings. On the one hand, the force of the premises blinds us to the fallacy of the reasoning; on the other, the soundness of the reasoning leads us to lose sight of the falsity of the premises. Now, the primary principles of Mathematics are clear and certain, and stamp the conviction of their realty as soon as comprehended by the mind; and no proposition is admitted unless its certainty is clearly recognized as a first principle, or as a clearly established truth. The mind is trained to the examination of premises. Even in the elementary branches, a vigorous exercise of mental power is needed to fully apprehend the data; while the first principles of the calculus and higher branches of pure Mathematics, are products of a very high abstraction, and, the ultimate propositions and the trains of reasoning involved, demand the exercise of great intellectual power. Are the fundamental propositions of Applied Mathematics, which are rendered more complex by the union of conceptions from physical laws with the difficult abstractions of pure Mathematics—*passively* received? Dynamics, Optics, Acoustics, Astronomy, Electricity and other sciences in which analysis reaches its highest applications--do their principles condemn to mere "mental inertion"—their highest development to an absolute "minimum" of thought? Hence it appears that mathematics exact the critical examination of data as a necessary condition of conquering their difficulties; we *must* concentrate our attention on first principles till these are fully comprehended and become genuine elements of knowledge; thus trained we acquire—not a "blind credulity" but a *habit of caution* in the admission of premises.

But if their utility is great in guarding us against errors in data, it is still greater in fortifying us against fallacies in *reasoning*. On this point little need be added to what has already been advanced. There appears to be in the human mind a natural tendency to perceive resemblances where none exist, and to be led astray by false analogies. Hence the necessity of caution in admitting the connection between the successive steps in any argument. Now, granting for a moment that mathematics preclude the possibility of sophistry in thought—tolerate no false analogy from deceptive resemblances—the successive steps in their processes must be immediately comprehended as necessary. Hence the mind becomes habituated to the evident connection between them and hesitates to admit their validity when it does not clearly perceive their relation. Is there not thus formed a habit of caution which is of the highest importance in the reasonings of experience? If we refuse to sanction any step in the reasoning till we clearly comprehend its logical connection with the preceding one, do we not adopt the surest possible safeguard against a fruitful source of error?

But, as before stated, I do not believe that mental sophistries are excluded from mathematical reasoning. Owing to the abstruseness of the conceptions employed, there is danger of including something irrelevant, excluding something comprehended, and supposing an analogy where none exists. And since these fallacies occur in spite of a *rigorous method*, they must be such as arise from the admission of false premises or propositions; and the frequency of their occurrence, and of their discovery and elimination, must develope a *habit* of caution in the examination of connecting propositions till their relevancy is plainly seen.

I think, then, I am justified in maintaining the value of mathematics as imparting habits of caution in the admission of premises and intermediate principles. And yet it has been asserted that their tendency is to develope a blind credulity and an uncompromising scepticism! If any Mathematitian has exhibited a blind credulity in the admission of erroneous data and the deduction of extravagant conclusions, it must have been *in spite* of his mathematical training, and not in consequence of it. The *post hoc, ergo propter hoc* style of argument has been a common weapon with the speculative opponents of mathematical discipline. Mathematicians have sometimes proved unfortunate in the management of their business affairs, and forth with mathematical discipline is charged with the failure, and pronounced to disquality for the affairs of life and for common reasoning. But, I suspect we know of many failures which cannot possibly be traced to the influence of mathematics. Despite the caution and sagacity constantly required in their own science, they have sometimes been too prone to manifest a 'facile credence' in the reception of principles and theories which rested mainly on the authority of their originators and supporters; but it would not be difficult to find illustrations of "facile credence" that can hardly be traced to the influence of mathematics. Mathematical metaphysicians have occasionally been guilty of absurd theories in metaphysics; but why should mathematics rather than metaphysics, be held responsible for the absurdities? Would it not be well to consider the legions of *non*—mathematical metaphysicians who have been guilty of equal or still greater absurdities? The history of metaphysics thus far is a history of mental aberration; are mathematics responsible for the ceaseless recurrence of erroneous systems? The great modern champion of the paramount importance of metaphysical research admits that the "past history of philosophy has, in a great measure, been only a history of variation and error"—have mathematics been the cause of this endless uncertainty?

As to scepticism, I suppose that there is some ground for the long-standing complaint against mathematicians, that they are hard to convince "But it is a far greater disqualification both for philosophy and for the affairs of life to be too easily convinced; to have too low a standard of proof. The only sound intellects are those which in the first instance set their standard of proof high. Practice in concrete affairs soon teaches them to make the necessary abatement; but they retain the consciousness without which there is no sound, practical reasoning, that in accepting inferior evidence because there is none better to be had, they do not, by that acceptance raise it to completeness."

3. But not only do mathematics educate to the use of correct forms of reasoning, and sagacity in the discovery and correction of fallacies, they educe a general vigor and comprehension of thought which still further prepare the mind for every kind of logical investigation. In support of this propositi on but little more need be advanced as I have already shown heir beneficial influence in expanding and strengthening the several mental powers. The first principles of mathematics—especially of the higher branches—though universal and necessary truths are not *passively* received but exact a conscious activity of mind for their clear apprehension; while the constant exercise in discerning the relations of truths so abstract and comprehensive, tends to the highest development of the intellectual powers. And the application of mathematics to physical laws, necessitates a grasp of mind still more comprehensive; for with the difficult abstractions of the pure mathematics are combined new conceptions from physical laws which increase the complexity of the data, the abstruseness of the connecting propositions and the consequent laboriousness of the trains of reasoning. Yet it has been said that mathematics call forth but a minimum of thought because the principles are self-evident, and every step in their reasonings are equally self evident, though the discovery of new truths may indicate a philosophic genius! Such an assertion could never have been uttered by any one possessing a knowledge of the subject beyond its most elementary principles. If by self-evident principles be meant such as are *passively* received by the mind, then mathematical principles, even in the mere elements of the science, are not *self-evident*; and still less the propositions employed in the demonstrations. The fundamental principles of abstract mathematics strike the mind with the conviction of their certainty *as soon as they are understood*; and the successive steps of a mathematical demonstration are equally self-evident *as soon as their relation is clearly comprehended*. But, as already shown, a vigorous exercise of intellect is required, especially in the higher mathematics, to understand the necessary data, and to comprehend the logical relation of the several propositions, before their *self-evident* nature is viewed in their *necessity* and *universality*.

Is there no energy of thought required to comprehend the successive steps of the demonstrations in the sublime geometry of Newton? The eleventh section of his Principia has been pronounced by a great philosopher to be characterized by "a spirit of far-reaching thought which distinguishes it beyond any other production of the human intellect"—does it require only a minimun af thought to understand his reasonings and to grasp, in all its comprehensiveness the fruitfulness of the results? By the application of analysis the complicated dynamics of the solar system are brought within reach of the human intellect—do the investigations determine thought to its "feeblest development?"

Nor is it true that though original discoveries and inventions require a

CONTINUED PAGE 18.)