and that it is not close to either the water surface or the bottom of the tank. It may therefore be assumed that the water converges toward the orifice from a hemisphere of infinite radius to one of diameter $A\,B$. On that assumption we will compute the excess of pressure on the left hand over that on the right-hand side of the vessel, due to the opening $A\,B$ of the orifice.

The pressure decrease at any point on the right, outside of the area $A\,B$ of the orifice is equal to the velocity head existing at that point. The summation of these decreases in pressure over the right-hand face is as follows, taking $A\,B = 2r$:—

$$\int_{\infty}^{\pi r^2} p\,dF = \int_{\infty}^{\pi r^2} \frac{v^2\gamma\,dF}{2g} = \int_{\infty}^{r} \frac{Q\gamma}{8\pi^2 r^4 g} \;.\; 2\pi v dr = \frac{Q^2\gamma}{4\pi g}$$

$$\int_{\infty} \frac{vdr}{r^3}\,dr = \frac{Q^2\gamma}{8\pi r^2 g}$$

Then, applying to this problem the same methods used in obtaining Equation (5) in the preceding problem, we have

$$Fh\gamma + \frac{Q^2\gamma}{8\pi r^2 g} = \frac{cF\gamma v^2}{g} \quad . \quad . \quad (6)$$

Placing $Q = cFv$, $v = \phi\sqrt{2gh}$ where ϕ is an *experimental* coefficient of velocity or friction, and $F = \pi r^2$, the equation finally reduces to

$$1 + \frac{c^2\phi^2}{4} = 2c\phi^2$$

from which for $\phi = 0.95$, we find $c = 0.60$, which agrees well with the results of experiments on such orifices.

As the orifice becomes large with respect to the size of the vessel, or is near the bottom or a corner, the mathematical analysis of the problem becomes much more complicated. Newton's law, however, is seen to present a rational theoretical basis for the determination of coefficients of contraction.

The second phase of the application of Newton's second law is found chiefly in hydraulic problems involving a sudden change in the conditions of flow, such as sudden enlargement in a pipe flowing full of water, the standing wave, or the "bore" created by the sudden stoppage of flow in an open flume. It is somewhat the the same as the problem of "direct central impact" in the mechanics of solids. Such problems can be attacked either by the method of "work and energy" or by the "momentum" or "impact" method. It will be shown that the method of "work and energy" seems to lead to erroneous conclusions and that apparently the "impact" method should be used in such cases.

Before proceeding it is well to recall that the method of "work and energy" is based upon the following differential equations:—

$$vdv = ads \quad . \quad . \quad . \quad (7),$$

a purely mathematical equation, and

$$\frac{dv}{dt} = a = \frac{P}{M} \quad . \quad . \quad . \quad (8),$$

an incomplete form of Newton's second law, the differential equation through elimination of "a", becoming

$$Mvdv = Pds \quad . \quad . \quad . \quad (9);$$

which equation, when integrated between the limits of v_1 and v_2 for velocity and s_1 and s_2 for distance which correspond to the limits O and t for time, becomes for constant values of M and P,

$$\frac{M\,(v_2^2 - v_1^2)}{2} = P\,(s_2 - s_1) \quad . \quad (9a)$$

On the other hand, in the case of a sudden change in the conditions of flow, the change of momentum occurs *not* as a small change in the velocity for the finite mass, M, *but* as a large change in the velocity, $v = (v_2 - v_1)$ for a small amount of mass, dM, in the time, dt. Hence the method of work and energy which is based upon the first mentioned type of change of momentum cannot correctly apply to a problem where the conditions are such that a change of momentum of the second type exists. The "momentum" or "impact" equation, which should be used, is

$$v\frac{dM}{dt} = P \quad . \quad . \quad . \quad (10)$$

or, since $v = v_2 - v_1$ and $\frac{dM}{dt} = \frac{Q\gamma}{g}$

$$\frac{Q\gamma\,(v_2 - v_1)}{g} = P \quad . \quad . \quad . \quad (10a)$$

We will first consider the problem of an abrupt enlargement in cross-section of a pipe flowing full of water,

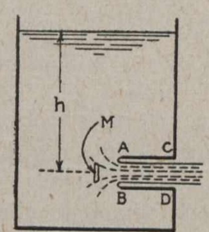
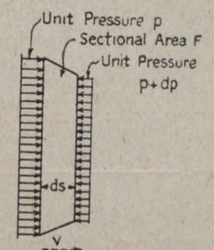


Figure 1
Re-entrant Mouthpiece



Enlarged View of Elementary Mass

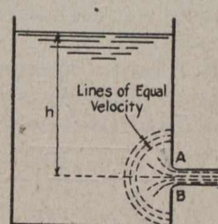


Figure 2
Sharp Edged Circular Orifice

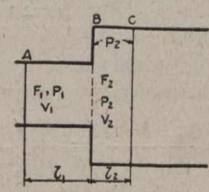


Figure 3
Sudden Enlargement in Pipe

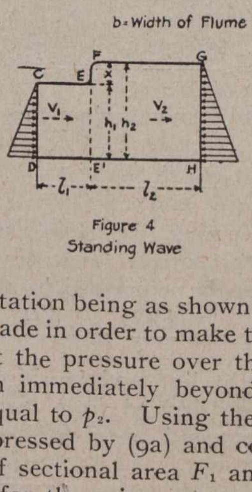


Figure 4
Standing Wave

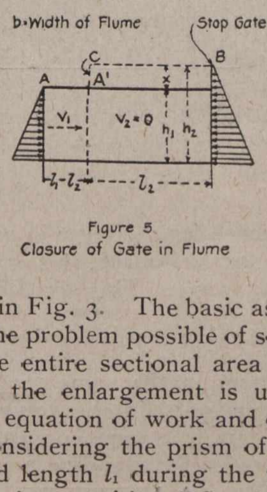


Figure 5
Closure of Gate in Flume

the notation being as shown in Fig. 3. The basic assumption made in order to make the problem possible of solution is that the pressure over the entire sectional area of the stream immediately beyond the enlargement is uniform and equal to p_2. Using the equation of work and energy as expressed by (9a) and considering the prism of water AB, of sectional area F_1 and length l_1 during the time it takes for the prism to come into position BC, where its sectional area is F_2 and length $l_2 = \frac{F_1}{F_2}l_1$, we obtain the following result:—

$$\frac{F_1 l_1 \gamma\,(v_2^2 - v_1^2)}{2g} = F_1 p_1 l_1 - F_2 p_2 l_2$$

or

$$\frac{p_1 - p_2}{\gamma} = \frac{v_2^2 - v_1^2}{2g} \quad . \quad . \quad (11)$$

This equation indicates no loss of head at the abrupt enlargement due to shock or impact and is shown by experiment to be incorrect.