11. (1) If there are n geometric means, then q (if $>p$)$=pr^{n+1}$, when r is common ratio,

$$\therefore \left(\frac{q}{p}\right)^{\frac{1}{n+1}}=r.$$

$$\therefore \text{1st mean} = p\,.\left(\frac{q}{p}\right)^{\frac{1}{n+1}},$$

$$\text{2nd mean} = p\,.\left(\frac{q}{p}\right)^{\frac{2}{n+1}},$$

$$\text{3rd mean} = p\,.\left(\frac{q}{p}\right)^{\frac{3}{n+1}},$$

$$\text{and } n\text{th mean} = p\left(\frac{q}{p}\right)^{\frac{n}{n+1}},$$

$$\therefore \text{product of mean} = p^n\left(\frac{q}{p}\right)^{\frac{1+2+3..+n}{n+1}}$$

$$=p^n\left(\frac{q}{p}\right)^{\frac{n(n+1)}{2(n+1)}}$$

$$=p^n\left(\frac{q}{p}\right)^{\frac{n}{2}}$$

$$=p^n \times \frac{q^{\frac{n}{2}}}{p^{\frac{n}{2}}}=(pq)^{\frac{n}{2}}.$$

(2) Let $y = xr$, $z = xr^2$, $\therefore\ x^3y^3z^3$

$$\left(\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{z^3}\right) = x^9r^9\left(\frac{1}{x^3}+\frac{1}{x^3r^3}+\frac{1}{x^3r^6}\right)$$

$$=x^9r^9\left(\frac{r^6+r^3+1}{x^3r^6}\right)=(r^6+r^3+1)x^6$$

$$=x^6r^6+x^6r^3+x^6$$

$$=z^3+y^3+x^3\text{—Q.E.D.}$$

12. (1) Give a proof of Binomial Theorem for a positive index.

(2) Write down the coefficient of x^{2r+1} in the expansion of $\left(x-\frac{1}{x}\right)^{2n+1}$

12. (1) Book-work.

(2) $\left(x-\frac{1}{x}\right)^{2n+1} = x^{2n+1}\left(1-\frac{1}{x^2}\right)^{2n+1}$

To find coefficient of x^{2r+1} we must find coefficient of $x^{2r+1-(2n+1)}$, or $x^{2(r-n)}$ in $\left(1-\frac{1}{x^2}\right)^{2n+1}$, which is the same thing as finding coefficient of x^{n-r} in $(1-x)^{2n+1}$,

$$\therefore \text{coefficient}=\frac{\lfloor 2n+1}{\lfloor n-r\,\lfloor n+r+1}.$$

13. (1) Expand $a\sqrt{\left\{1-\frac{x^2}{a^2}\right\}}$ to 4 terms.

(2) Find the sum of the squares of the coefficients in the expansion of $(1+x)^n$, where n is a positive integer.

13. (1) $a\sqrt{1-\frac{x^2}{a^2}}=a\left\{1-\frac{x^2}{a^2}\right\}^{\frac{1}{2}}$

$$=a\left\{1-\frac{x^2}{2a^2}-\frac{1}{8}\,\frac{x^4}{a^4}-\frac{1}{16}\,\frac{x^6}{a^6}\cdots\cdots\right\}.$$

(2) $(1+x)^n=1+nx+\frac{n(n-1)}{\lfloor 2}x^2+\ldots+x^n$ (1)

$(x+1)^n=x^n+nx^{n-1}+\frac{n(n-1)}{\lfloor 2}x^{n-2}+\ldots\ldots+1$ (2)

$\therefore\ (1+x)^{2n}=(1)\times(2).$

Selecting the coefficient of x^n in product of (1) and (2) we find it to be sum required,

but coefficient of x^n in $(1+x)^{2n}$ is $\frac{\lfloor 2n}{\lfloor n\,\lfloor n}$,

$\therefore$ sum of squares of coefficients $=\frac{\lfloor 2n}{\lfloor n\,\lfloor n}$.

MODERN LANGUAGES.

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EXERCISES IN ENGLISH.

The first five questions are based on the lesson, "Ocean," in the Fourth Reader.

1. Point out the phrases in the following, and tell their grammatical value and relation.

(*a*) Ten thousand fleets sweep over thee in vain.

(*b*) Man marks the earth with ruin.

(*c*) Nor doth remain a shadow of man's ravage.

(*d*) Spurning him from thy bosom to the skies.

(*e*) From out thy slime the monsters of the deep are made.

2. Substitute for the following words or phrases others of equivalent meaning:—unknelled, haply, quake, arbiter, mar, realms, azure, glasses, torrid clime, from a boy, wantoned with thy breakers.

3. Explain the force of the italicized words in the following:—Sweep over thee *in vain*; *thunderstrike* the walls, oak *leviathan*, *clay*