UNIVERSITY WORK.

MATHEMATICS.

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SOLUTIONS OF CAMBRIDGE PROBLEMS.

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5. If $\phi(x, n)=\frac{1}{x}-n\cdot\frac{1}{x+1}+\frac{n(n-1)}{\lfloor\underline{2}}\cdot\frac{1}{x+2}-$, etc.

where n is a positive integer, find a relation connecting $\phi(x, n)$ and $\phi(x+1, n+1)$; and thence show that $\phi(x, n)=\frac{n!(x-1)!}{(x+n)!}$,

we have $\phi(n, n)=\frac{1}{x}-n\cdot\frac{1}{x+1}+\frac{n(n-1)}{\lfloor\underline{2}}\cdot\frac{1}{x+2}-$ etc.,

$$=\frac{1}{x}\left\{1-n\cdot\frac{x}{x+1}+\frac{n(n-1)}{\lfloor\underline{2}}\cdot\frac{x}{x+2}-\text{ etc.,}\right\}$$

$$=\frac{1}{x}\left\{1-n\left(1-\frac{1}{x+1}\right)+\frac{n(n-1)}{\lfloor\underline{2}}\left(1-\frac{2}{x+2}\right)-\text{ etc.,}\right\}$$

$$=\frac{1}{x}\left\{1-n+\frac{n(n-1)}{\lfloor\underline{2}}-\text{ etc.,}+\frac{n}{x+1}-\frac{n(n-1)}{\lfloor\underline{2}}\cdot\frac{2}{x+2}+\text{ etc.,}\right\}$$

$$=\frac{n}{x}\left\{\frac{1}{x+1}-(n-1)\frac{1}{x+2}+\frac{(n-1)(n-2)}{\lfloor\underline{2}}\quad\frac{1}{x+3}-\text{ etc.,}\right\}$$

$$\left(\because 1-n+\frac{n(n-1)}{\lfloor\underline{2}}-\text{etc.,}=(1-1)^{n}=0\right),$$

$$i.e., \phi(x, n)=\frac{n}{x}\ \phi(x+1, n-1),$$

$$\phi(x+1, n-1)=\frac{n-1}{x+1}\ \phi(x+2, n-2),$$

etc. = etc.

$$\phi(x+n-1, n-\overline{n-1})=\frac{1}{x+n-1}\phi(x+n, 0)$$
$$=\frac{1}{x+n-1}\cdot\frac{1}{x+n},$$

multiplying and cancelling

$$\phi(x, n)=\frac{n!(x-1)!}{(x+n)!}.\quad \text{Q.E.D.}$$

6. From the identity $x^3+1=(x+1)(x^2-x+1)$, show that if m be a positive integer

$$1-\frac{6m-2}{1.2}+\frac{(6m-3)(6m-4)}{1.2.3}-\frac{(6m-4)(6m-5)(6m-6)}{1.2.3.4}+\ldots=0.$$

Taking logs of both sides

$$x^3-\frac{x^6}{2}+\frac{x^9}{3}-\frac{x^{12}}{4}+\ldots$$
$$=x-\frac{x^2}{2}+\frac{x^3}{3}-\text{ etc.,}$$
$$-\left\{x(1-x)+\frac{x^2(1-x)^2}{2}+\frac{x^3(1-x)^3}{3}+\ldots\right\},$$

Equate co-efficients of x^{6m+1}, then

$$0=\frac{1}{6m+1}-\left\{\frac{1}{6m+1}-\frac{6m}{6m}+\frac{(6m-1)(6m-2)}{1.2},\ \frac{1}{6m-1}-\frac{(6m-2)(6m-3)(6m-4)}{1.2.3},\ \frac{1}{6m-2}+\text{etc.,}\right\}$$

which is readily simplified to

$$1-\frac{6m-2}{1.2}+\frac{(6m-3)(6m-4)}{1.2.3}-\text{etc.}=0.$$

Q.E.D.

SOLUTIONS OF SELECTED PROBLEMS.

See April No., 1883.

1. If $a+b+c=0$, prove

$$\left(\frac{b^3-c^3}{a^3}+\frac{c^3-a^3}{b^3}+\frac{a^3-b^3}{c^3}\right)\left(\frac{a^3}{b^3-c^3}+\frac{b^3}{c^3-a^3}+\frac{c^3}{a^3-b^3}\right)$$
$$=3b-4(a^3+b^3+c^3)(a^{-3}+b^{-3}+c^{-3}),$$