

$$\therefore AA' \cdot AA'' = \sqrt{\left(\frac{a^2 + b^2 + c^2}{2}\right)^2 - \frac{3}{4} S^2}$$

$$= \sqrt{a^4 + b^4 + c^4 - b^2 c^2 - c^2 a^2 - a^2 b^2}$$

a symmetrical result.

XIV. If (p, q, r) be the perpendiculars on the sides of a triangle ABC from the centre of the circumscribing circle, prove that

$$agr + brp + cpq = \frac{abc}{4}.$$

$$p = \frac{a}{2 \tan A}, \text{ \&c.}$$

Identity becomes

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

XV. A circle is described through the foci of an ellipse and any point on its circumference. Two tangents are drawn to this circle through one extremity of the major axis. Shew that the locus of the points of contact of these tangents is a circle whose radius is equal to the minor axis of the ellipse.

Let AP, AP' be a pair of tangents;

$$\text{Then } AP^2 = AP'^2 = AS \cdot AS'$$

$$= (a - ae)(a + ae) = b^2.$$

Wherefore, \&c.

16. CP, CD are conjugate semi-axes of an ellipse; PNE is drawn parallel to the minor axis CB , meeting the major axis in N and CD in E . Prove that the area of the triangle

$$PCE \text{ is } = 2CB^2 \cdot \frac{PN}{CN}.$$

The Examiner requests us to state that there is an error in the enunciation; the result should be as under.

$$\text{Area} = \frac{1}{2} PE \cdot CN. \quad PE = \frac{b^2}{y}, \quad CN = x$$

$$= \frac{1}{2} b^2 \cdot \frac{x}{y} = \frac{1}{2} BC^2 \cdot \frac{x}{y}.$$

18. TP, TQ are two tangents to an ellipse at right angles to one another, S a focus, prove that

$$\sin^2 SPT + \sin^2 SQT = \text{constant}.$$

Comparing the equations to PT

$$\frac{xx^1}{a^2} + \frac{yy^1}{b^2} = 1, \text{ and}$$

$$x \cos a + y \sin a = \sqrt{a^2 \cos^2 a + b^2 \sin^2 a}$$

$$\frac{x^1}{a^2 \cos a} = \frac{y^1}{b^2 \sin a} = \frac{1}{\sqrt{a^2 \cos^2 a + b^2 \sin^2 a}}.$$

If r be radius vector to point of contact, and p perpendicular from focus on tangent,

$$\frac{p}{r} = \sin SPT$$

$$= \frac{\sqrt{a^2 \cos^2 a + b^2 \sin^2 a} - ae \cos a}{a - ex^1}$$

$$= \frac{\sqrt{a^2 \cos^2 a + b^2 \sin^2 a}}{a},$$

whence sum of squares of sines

$$= \frac{a^2 + b^2}{a^2}, \text{ a constant.}$$

Solution of problem No. 7, p. 143, Elementary Arithmetic (Smith & MacMurchy), by A. E. Moore, teacher.

£	s.	d.	
184	11	6	av'ge for 11 yrs., 1853-63
			11

2030	6	6	aggregate gain, 1853-63
76	8	4	loss in year 1853

2106	14	10	aggregate gain, 1854-63
151	9	10	= gain in 1863

$$11)2258 \quad 4 \quad 8 = \text{aggregate gain, 1854-64}$$

$$\text{£}205 \quad 5 \quad 10\frac{1}{2} = \text{average for the 11 years from 1854 to 1864.} - \text{Ans.}$$

The answer given in the text is £191 8s.; it should be as above.