MATHEMATICS.

PROBLEM PAPER JUNIOR MATRICULATION, 1879.

Solved by G. Ross, Mathematical Scholar, Toronto University.)

1. An A.P., a G.P., and an H.P. have each the same first and last terms, and the same number of terms (n), and the rth terms are a_j, b_j, c_j; prove that

$$a_{j+1} : b_{j+1} = b_{n-j} : c_{n-j}$$

Let p be first term and l last term.

Let g be com. diff. in A.P. and f be com. ratio in G P.

$$\therefore g = \frac{l-p}{n-1} \qquad f = \left(\frac{l}{p}\right)^{\frac{1}{n-1}}$$

and $$a_{j+1} = \frac{p(n-1)+j(l-p)}{n-1}$$

$$b_{j+1} = p\left(\frac{l}{p}\right)^{\frac{j}{n+1}}$$

$$c_{n-i} = p\left(\frac{l}{p}\right)^{\frac{n-j-1}{n-1}}$$

$$c_{n-j} = \frac{pl(n-1)}{l(n-1)+(n-j-1)(p-l)}$$

and $$\frac{a_{j+1}}{b_{j+1}} = \frac{p(n-1)+j(l-p)}{(n-1)p\left(\frac{l}{p}\right)^{\frac{j}{n-1}}}$$

$$\frac{b_{n-j}}{c_{n-j}} = \frac{p(n-1)+j(l-p)}{(n-1)p\left(\frac{l}{p}\right)^{\frac{j}{n-1}}} \text{ \&c.}$$

2. If p be nearly equal to q.

$\dfrac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$ is a close approximation to $\left(\dfrac{p}{q}\right)^{\frac{1}{n}}$. Let $\left(\dfrac{p}{q}\right)^{\frac{1}{n}} = 1+x$

$$\therefore \frac{p}{q} = (1+x)^n = 1+nx+\frac{n(n-1)x^2}{|2}$$

taking the first two terms of the series we have

$$x = \frac{p-q}{nq}$$

again, substituting this value in the above, we have

$$\frac{p}{q} = 1+nx\left(1+\frac{n-1}{2}\,\frac{p-q}{nq}\right)$$

$$\therefore x = \frac{2(p-q)}{n(p+q)-(p-q)}$$

$$\therefore \left(\frac{p}{q}\right)^{\frac{1}{n}} = 1+\frac{2(p-q)}{n(p+q)-(p-q)}$$

$$= \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$$

and if $\left(\frac{p}{q}\right)$ differ from 1 only in the $(r+1)^{th}$ decimal place, this approximation will be correct to $2r$ places. For, in finding the first value of x we discarded all the quantities involving n^2 and higher powers, that is all the quantities beginning at the $(2r+1)^{th}$ decimal place: hence, this result is correct to $2r$ places.

3. Having given

$$yz + \frac{1}{yz} - ax - \frac{b}{x} = zx + \frac{1}{zx} - ay - \frac{b}{y}$$

$$= xy + \frac{1}{xy} - az - \frac{b}{z}$$

prove that if x, y, z be all equal, $ab=1$ and that each member of this equation is equal to zero.

Taking 1st and 2nd members of this equation together and dividing by $(y-x)$, we have

$$z - \frac{1}{xyz} + a - \frac{b}{xy} = 0 \qquad (4)$$

and taking the 1st and 3rd together

$$y - \frac{1}{xyz} + a - \frac{b}{xz} = 0 \qquad (5)$$

$\therefore$ from (4) and (5) $b = xyz$