365. PROBLEM. *To transform an equation so that the roots shall be multiplied by a given factor m.*

Solution. Since the roots are to be multiplied by m, the new unknown quantity must be equal to mx. So if we call this quantity y, we have

$$y = mx,$$

which gives

$$x = \frac{y}{m}.$$

Substituting this in the general equation, it becomes

$$\frac{y^n}{m^n} + p_1\frac{y^{n-1}}{m^{n-1}} + p_2\frac{y^{n-2}}{m^{n-2}} + \ldots + p_n = 0.$$

Multiplying all the terms by m^n, the equation becomes

$$y^n + mp_1y^{n-1} + m^2p_2y^{n-2} + \ldots + m^np_n = 0.$$

Hence the rule,

Multiply the coefficient of the second term by m, that of the third by m^2, and so on to the last term, which will be multiplied by m^n.

If the roots are to be divided, we divide the terms in the same order.

EXERCISES.

1. Make the roots of $x^2 - 2x + 3 = 0$ four times as great.
2. Divide the same roots by 2.

366. PROBLEM. *To transform an equation so that its roots shall be squared.*

Solution. Let the given equation be

$$x^4 + p_1x^3 + p_2x^2 + p_3x + p_4 = 0.$$

If y be the unknown quantity of the new equation, we must have

$$y = x^2,$$

which gives

$$x = \pm y^{\frac{1}{2}}.$$

If we substitute $x = y^{\frac{1}{2}}$ in the given equation, it may be reduced to the form

$$y^2 + p_2y + p_4 + (p_1y + p_3)\,y^{\frac{1}{2}} = 0.$$