$$\text{or} \qquad q_1 = \frac{C_1 q_2}{C_2} + C_1 R_2 \frac{dq_2}{dt}$$

We may differentiate this equation and substitute in (1), obtaining

$$\frac{d^3q_2}{dt^2} + \frac{C_1R_1R_2 + L + \dfrac{LC_1}{C_2}}{L\,C_1\,R_2}\frac{d^2q_2}{dt^2} + \frac{R_1 + R_2 + R_1\dfrac{C_1}{C_2}}{L\,C_1\,R_2}\frac{dq_2}{dt} + \frac{q_2}{LC_1C_2R_2} = 0 \tag{3}$$

We may apply the general method for linear differential equations with constant co-efficients by placing $q_2 = \epsilon^{\lambda t}$, substitution of which in (3) gives

$$\lambda^3 + \frac{C_1R_1R_2 + L + L\dfrac{C_1}{C_2}}{L\,C_1\,R_2}\lambda^2 + \frac{R_1 + R_2 + R_1\dfrac{C_1}{C_2}}{L\,C_1\,R_2}\lambda + \frac{1}{L\,C_1\,C_2\,R_2} = 0 \tag{4}$$

yielding three values of λ, so that

$$q_2 = a_1\epsilon^{\lambda_1 t} + a_2\,\epsilon^{\lambda_2 t} + a_3\epsilon^{\lambda_3 t} \tag{5}$$

It will be shown later that only one root of the λ equation is real. The other two must be conjugate imaginaries. We may call $\lambda_1 = \lambda$, the real root, and the other two may be written $\alpha + i\beta$ and $\alpha - i\beta$. So we have finally

$$q_2 = a\epsilon^{\lambda t} + \epsilon^{\alpha t}\,(A\cos\beta t + B\sin\beta t) \tag{6}$$

Differentiation and substitution in (2) give

$$q_1 = a\left(\frac{C_1}{C_2} + C_1R_2\,\lambda\right)\epsilon^{\lambda t} \tag{7}$$

$$+\ \epsilon^{\alpha t}\left[\left\{\frac{C_1}{C_2}A + C_1R_2(A\alpha + B\beta)\right\}\cos\beta t + \left\{\frac{C_1}{C_2}B + C_1R_2(B\alpha - A\beta)\right\}\sin\beta t\right]$$

We may, of course, write (6) as

$$q_2 = a\epsilon^{\lambda t} + A\epsilon^{\alpha t}\cos\,(\beta t - \phi) \tag{8}$$

$$\text{where}\quad A = \sqrt{A^2 + B^2}\ \text{and}\ \phi = \tan^{-1}\frac{B}{A} \tag{9}$$

and in the same manner

$$q_1 = \left(\frac{C_1}{C_2} + C_1R_2\,\lambda\right)a\,e^{\lambda t} + A_1\,e^{\alpha t}\cos\,(\beta t - \phi + \omega) \tag{10}$$

$$\text{where} \qquad A_1 = A\,\sqrt{\left(\frac{C_1}{C_2} + C_1\,R_2\,\alpha\right)^2 + C_1^2\,R_2^2\,\beta^2}$$

$$\text{and} \quad \omega = \tan^{-1}\frac{C_2\,R_2\,\beta}{1 - C_2\,R_2\,\alpha} \tag{11}$$

ω is the difference in phase of the two discharges. It will be noticed that both discharges are oscillatory and consist in each case of a damped oscillation impressed upon an exponential discharge.