

so we have ten edges in all. Then we may pass a plane through each of the six edges of the tetrahedron and the new vertex giving six new planes or faces, making ten in all. Also from the new vertex we have a tetrahedron resting on each base of the old tetrahedron, so we have five tetrahedrons in all. Collecting the members of this family we have—

1 hypertetrahedron,
5 tetrahedrons,
10 faces
10 lines,
5 points.

We might carry this part of the discussion further, but I will merely add in passing that just as there are five regular polyhedrons in three-fold space, there are six regular hypersolids in four-fold space.

We may note some peculiarities of four-fold space by considering how space of different dimensions can be cut in two parts. Space of one dimension or the straight line can be divided by a point on the line. In order to get from one side to the other, the point must be passed, i.e. the point constitutes a barrier in one dimensional space. In the same way, the straight line constitutes a barrier in two dimensional space, and the plane is a barrier in three-fold space. In each of the foregoing cases, the barrier is a space of one dimension less than the space it divides. Following out the analogy, four-fold space can be separated into two parts only by a complete three-fold space.

Again, in the case of the line divided by the point, the barrier may be passed easily if two dimensional movement is allowed, i.e. we may move out of the line around the point and back again. In the same way a linear barrier in two-fold space is easily passed by recourse to a third direction of motion. Finally, a plane may be passed by recourse to the fourth dimensional motion and a point may be taken around the infinite plane without passing through it.

Suppose we look for a moment at the matter of rotation of a figure in any of these spaces. In one dimensional space no rotation is possible. In a plane, any figure may rotate about any fixed point in the plane, all points of the figure describing concentric circles. In three-fold space, any figure may be ro-