From the geometric relation between E, E_1 and E_2 (Fig. 52)

$E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \theta$ .... (22)

If θ represents the angle between E_1 and E, then

$\sin \theta / \sin \phi = E_2/E$ .... (23)

Example 15.—An alternating e.m.f. expressed by the equation $e = 60 \sin \omega t$ is generated in one coil of wire, and a second e.m.f., expressed by the equation $e = 80 \sin (\omega t - 30)$, is generated in another coil. If the two coils are connected in series, to determine the resultant e.m.f.,

The second e.m.f. lags 30° behind the first.

The maximum value of the resultant is

$E = \sqrt{(60^2 + 80^2 + 2 \times 60 \times 80 \times \cos 30)} = 135.$

If θ is the angle between the first e.m.f. and the resultant,

$\sin \theta = \sin 30 \times 80/135$, and $\theta = 17° \ 10'$.

Since the resultant e.m.f. lags θ° behind the first e.m.f., it will be expressed by the equation $e = 135 \sin (\omega t - 17° \ 10')$.

It has been noted in a previous chapter that when there is a change in the strength of the current in a circuit the accompanying change of magnetic flux induces in the circuit an e.m.f. which opposes the change of current strength. While this is of little or no importance in the case of a direct current, it is of very great importance in the case of alternating currents, the strength of which varies with great rapidity. The relation between the alternating e.m.f. and current in an inductive circuit is well illustrated by the mechanical analogy shown in Fig. 53. A crank pin C revolves with a uniform velocity and imparts a simple harmonic motion to a piston, which reciprocates in the cylinder A. The piston is composed of a rigid ring B surrounding a flexible diaphragm D. The two ends of the cylinder are connected externally by means of a long pipe, which corresponds to the external electric circuit. The alternating force transmitted to the piston corresponds to the alternating e.m.f. The cylinder

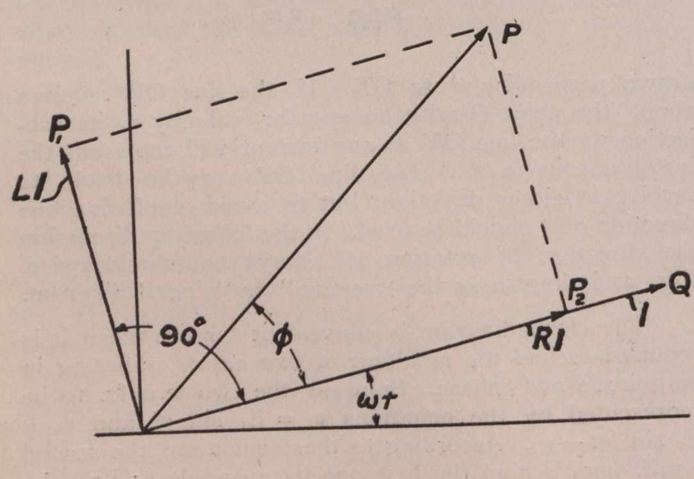

FIG. 54

and pipe are filled with any fluid medium, such as water or air. As the piston moves from right to left the inertia of the fluid will cause the diaphragm to extend, as shown in the figure, and when the piston reaches the end of its stroke the fluid will still be moving with considerable velocity, due to its momentum. The flexibility of the diaphragm will allow this motion to continue after the piston has started on the return stroke. The alternating motion of the fluid thus lags behind the alternating motion of the piston ring; i.e., there is a constant phase difference between them. The longer the pipe the greater the weight of fluid in motion and the greater the lag. If the frequency of the strokes of the piston is increased it will be more difficult for the fluid to follow, and there will consequently be a greater lag. If the pipe were frictionless and the diaphragm perfectly elastic, the motion of the fluid would be maximum when the piston reached the end of the stroke; i.e., there would be a lag of 90°,

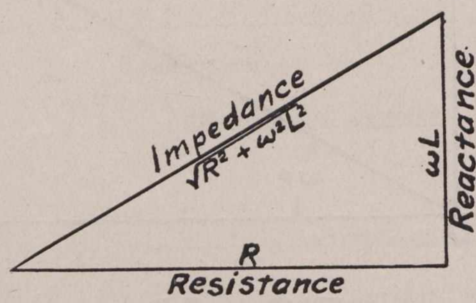

FIG. 55

or one-quarter phase. In this case the force impressed on the piston would be zero at the end of the stroke and maximum at the middle of the stroke, at which point the diaphragm would have maximum extension. The alternating force would thus be 90° ahead of the motion of the fluid. If, however, the pipe is not frictionless (which is always the case) a force must be applied to the piston to overcome the friction. This force will be maximum when the flow or fluid is maximum and zero when the flow is zero; i.e., it will be in phase with the flow of fluid and 90° behind the force required to overcome the inertia. The total force is made up of these two components.

The inductance of an electric circuit is analogous to the inertia of the fluid in the above illustration. It causes the current to lag behind the e.m.f. To overcome the inductance an e.m.f. 90° ahead of the current must be impressed on the circuit, and to overcome the resistance an e.m.f. in phase with the current is required. The total e.m.f. is the resultant of these two components, one 90° behind the other. The maximum value of the inductive component is ωLI, and the maximum value of the resistance component is RI, where f represents frequency, L inductance, R resistance, and $\omega = 2\pi f$. These components are combined by means of the vector diagram, as shown in Fig. 54. In this diagram OQ represents the current, OP_1 represents the inductance component, 90° ahead of the current, and OP_2 represents the resistance component, in phase with the current. From the geometry of the diagram the maximum value of the resultant e.m.f. is

$$E = \sqrt{(E^2 + E^2)} = I \sqrt{(R^2 + \omega^2 L^2)},$$

and $$I = \frac{E}{\sqrt{(R^2 + \omega^2 L^2)}} \text{ .... (24)}$$

It is also seen from the diagram that the current lags behind the e.m.f. by an angle ϕ such that

$$\tan \phi = \frac{\omega L}{R} \text{ .... (25)}$$

These two equations show that the effect of inductance is not only to make the current lag behind the e.m.f., but also to diminish its value. Equation (24) also indicates that the strength of the current diminishes to