The following demonstration includes both Props. V. and VI.: If there be two straight lines, sq. on greater is equal to the rectangle under the lines together with rectangle contained by greater and difference of lines by Prop. II. And rectangle under lines is equal to sq. on less with rectangle contained by less and difference by Prop. III. Hence sq. on greater is equal to sq. on less with rectangles contained by greater and difference and by less and difference; *i.e.* is equal to sq. on less with rectangle contained by sum and difference by Prop. I.; and this result is the first enunciation given above of Props. V. and VI.

Prop. VII. is the geometrical equivalent of the identity $(a-b)^2 = a^2 - 2ab + b^2$.

The following proof includes both IX. and X.: The sq. on AD (see above figures) is equal to the sqs. on BC, CD with twice the rectangle BC, CD. To each add the sq. on DB; then the sqs. on AD, DB are equal to the sqs. on BC, CD, DB with twice the rectangle BC, CD. And by Prop. VII. the sq. on DB with twice the rectangle BC, CD is equal to the sqs. on BC, CD. Hence the sqs. on AD, DB are equal to twice the sqs. on BC, CD.

Prop. XI. gives a geometrical construction for one of the roots of the quadratic equation $x^2 = a(a-x)$, a being the length of the given line, and x the part whose square is to be equal to the rectangle contained by the whole and the other part. A geometrical construction for the other root may be obtained as follows: Let AB be the given straight line. On it describe a sq. $ABCD$. Bisect AD in E. Produce EA to F making EF equal to EB. Produce BA to G. Bisect the angle GAF by AH. Let AH, BF produced meet in H. Draw HG perpendicular to BG. Complete the square AH. On GB describe the square $GKLB$. Produce BC to bisect GK in M. It may be shewn in quite the same way as in Prop. XI, that the square on AG is equal to the rectangle AB, BG; *i.e.* AB is externally divided in G so that the rectangle contained by the whole line and one of the parts is equal to the square on the other part. Hence AG must represent one of the roots of $x^2 = a(a-x)$, and AH in the ordinary figure representing the positive root, this must represent the negative root.

In the ordinary figure for this proposition, if from HA, HK be cut off equal to HB, AH is divided in medial section in K, for $\frac{AB}{AH} = \frac{AH}{HB} = \frac{AB-AH}{AH-HB}$, *i.e.* AH, $AK = HK^2$. If from KH, KL be cut off equal to KA, and if this process be carried on indefinitely, the point ultimately reached divides AB in medial section. For the distance from B ultimately reached is the sum *ad inf.* of the series $a - a\frac{-1+\sqrt{5}}{2} + a\left(\frac{-1+\sqrt{5}}{2}\right)^2 - \ldots$

$$= \frac{2a}{1+\sqrt{5}} = \frac{a}{2}(-1+\sqrt{5}) = AH.$$

Props. XII. and XIII. furnish us with a method of finding the area of a triangle when the three sides are given. For we have at once CD or BD; thence AD, and thence the area. (Prop. XLI., Bk. 1.)

The following paper was set at the Matriculation Examinations in June last. The solutions of the more difficult problems are given.

ALGEBRA.

HONOR.

1. If α, β be the roots of $x^2 + px + q = 0$, then $\alpha + \beta = -p$, $\alpha\beta = q$.

Form the equation whose roots are $\frac{1}{\alpha^3}$, $\frac{1}{\beta^3}$.

2. Solve the equations

$$z(x+y) = 3 + xy,$$
$$x(2y-1) = y,$$
$$xyz = 1.$$

3. A and B start to walk from two places M, N, at the same time, and towards one another. A is delayed one day on the road, in consequence of which he meets B 6 miles nearer M than he would otherwise have done. Continuing their walking, A and B reach N and M in $4\frac{2}{3}$ and $8\frac{3}{4}$ days respectively after leaving one another. Find the distance from M to N.

4. Is it a convention or a matter of proof that $a^{\frac{p}{q}}$ is the q^{th} root of the p^{th} power of a? Explain clearly.

Extract the square root of

$$a^{-\frac{3}{2}} - 2a^{-\frac{9}{8}} + 3a^{-\frac{3}{4}} - 2a^{-\frac{3}{8}} + 1.$$

Expand $\left(a^{\sqrt{3}} + a^{-\sqrt{3}}\right)^4$.

5. If b be not a perfect square, and $a^2 - b = c^2$, shew that

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+c}{2}} + \sqrt{\frac{a-c}{2}}.$$

Simplify $\frac{2+\sqrt{3}}{\sqrt{7-4\sqrt{3}}} - \frac{2-\sqrt{3}}{\sqrt{7+4\sqrt{3}}}$.

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, each of these fractions is equal to $\frac{ma+nc+pe}{mb+nd+pf}$.

Solve the equations—

$$\frac{x+2y}{3z} = \frac{z+2x}{3y} = \frac{y+2z}{3x} = x+y+z.$$

7. Find the sum to n terms of a Geometric series, whose first term and common ratio are given.

Sum to n terms the series $1 + 2x + 3x^2 + 4x^3 + \ldots$

An equilateral triangle is inscribed in a given circle, a circle within the triangle, an equilateral triangle in the second circle, and so on *ad. inf.*; compare the area of the first triangle with the sum of the areas of all that follow.

8. Find the Arithmetic, Geometric, and Harmonic means between a and b.

Show that three quantities cannot be at the same time in two kinds of progression.

9. Find the number of combinations of n different things r at a time.

An even number of points, n, are arranged at equal intervals on the circumference of a circle, and triangles are formed by joining them. Find the number of such that are oblique-angled.

10. Establish the Binomial Theorem in the case of a fractional index, assuming that it holds in the case of an integral index.

Find the n^{th} term in the expansion of

$$\left(1-\frac{1}{n}\right)^{1-\frac{1}{n}}$$

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1. $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{p}{q}$; $\therefore \frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{3}{\alpha\beta}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = -\frac{p^3}{q^3}$, $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{3pq-p^3}{q^3}$; and $\frac{1}{\alpha^3\beta^3} = \frac{1}{q^3}$; $\therefore$ eq. required is $x^2 + \frac{p^3-3pq}{q^3}x + \frac{1}{q^3} = 0$.

2. From (2) $\frac{1}{x} + \frac{1}{y} = 2$; from (1) and (3) $\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 3$; $\therefore z = -1$. Thence $x = -1 \pm \sqrt{2}$, $y = -1 \mp \sqrt{2}$.

3. Let x, y be the rates of A and B, and a the whole distance. Then from the first statement $\frac{x}{x+y}$ of $y = 6$; also $\frac{x}{x+y}(a-y) =$ distance travelled by A and to be travelled by B, $\frac{y}{x+y}(a-y) + y =$ distance travelled by B and to be travelled by A. $\therefore \frac{x}{x+y}$