The series then becomes, by giving n successive from 1 to n,

$$S=A\left\{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\ldots+\frac{1}{n+2}\right\}$$
$$+B\left\{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\ldots+\frac{1}{n+2}+\frac{1}{n+3}\right\}$$
$$+C\left\{\frac{1}{5}+\frac{1}{6}+\ldots+\frac{1}{n+3}+\frac{1}{n+4}\right\}$$
$$+D\left\{\frac{1}{6}+\ldots+\frac{1}{n+4}+\frac{1}{n+5}\right\}$$
$$=A\left(\frac{1}{3}\right)+(A+B)\frac{1}{4}+(A+B+C)\frac{1}{5}$$
$$+(B+C+D)\frac{1}{n+3}+(C+D)\frac{1}{n+4}+D\left(\frac{1}{n+5}\right)$$
$$=\frac{A}{3}+\frac{A+B}{4}+\frac{A+B+C}{5}$$
$$-\left\{\frac{A}{n+3}+\frac{A+B}{n+4}+\frac{A+B+C}{n+5}\right\}$$

because $A+B+C+D=0$.

Substituting the values of A, B, C and D this becomes

$$\frac{5}{36}-\frac{3n^2+15n+25}{3(n+3)(n+4)(n+5)}$$

when n is infinite this value is equal to $\frac{5}{36}$, because $\frac{A}{n+3}$, $\frac{A+B}{n+4}$ and $\frac{A+B+C}{n+5}$ are each equal to 0.

NOTE.—All the middle terms vanish because $A+B+C+D=0$; and this method can be applied to all those series the general term of whose numerator is at least two dimensions lower than the denominator, and when any term can be found from the preceding by changing n into $n+1$.

Proceeding in the same way with question 41,

$$\frac{11}{2.3.4.5.6}+\frac{35}{3.4.5.6.7}+\frac{81}{4.5.6.7.8}+\frac{155}{5.6.7.8.9},$$

etc., we see that the third differences are constant, and the general term becomes

$$\frac{n^3+5n^2+2n+3}{(n+1)(n+2)(n+3)(n+4)(n+5)}$$

$$A=\frac{5}{24},\ B=-\frac{44}{24},\ C=\frac{90}{24},\ D=-\frac{44}{24},\ E=-\frac{7}{24}$$

and as before, $A+B+C+D+E=0$.
and the series is

$$\frac{5}{24}\left\{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\cdots\frac{1}{n+1}\right\}$$
$$-\frac{44}{24}\left\{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\cdots\frac{1}{n+1}+\frac{1}{n+2}\right\}$$
$$+\frac{90}{24}\left\{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\cdots\frac{1}{n+2}+\frac{1}{n+3}\right\}$$
$$-\frac{44}{24}\left\{\frac{1}{5}+\frac{1}{6}\cdots\frac{1}{n+3}+\frac{1}{n+4}\right\}$$
$$-\frac{7}{24}\left\{\frac{1}{6}\cdots\frac{1}{n+4}+\frac{1}{n+5}\right\}$$
$$S=\frac{5}{24}\left(\frac{1}{2}\right)+\frac{1}{24}(5-44)\left(\frac{1}{3}\right)+\frac{1}{24}(5-44+90)$$
$$\left(\frac{1}{4}\right)+\frac{1}{24}(5-44+90-44)\left(\frac{1}{5}\right)$$
$$-\left\{\frac{5}{24}\left(\frac{1}{n+2}\right)+\frac{1}{24}(5-44)\left(\frac{1}{n+3}\right)\right.$$
$$+\frac{1}{24}(5-44+90)\left(\frac{1}{n+4}\right)$$
$$\left.+\frac{1}{24}(5-44+90-44)\left(\frac{1}{n+5}\right)\right\}$$

The intermediate terms all vanishing, this readily reduces to

$$\frac{73}{480}-\frac{4n^3+34n^2+86n+73}{4(n+2)(n+3)(n+4)(n+5)},$$

the value when n is infinite being $\frac{73}{480}$.

These results can be easily tested in both cases by making $n=0$, 1, 2, 3, 4, etc.

For this problem

$n=0$ gives $S=0$,

$n=1$ $\quad S=\frac{73}{180}$,

$n=2$ $\quad S=\frac{7}{240}$, &c.

In the first problem we have $n=0$ $S=0$,

$$n=1\ S=\frac{7}{3.4.5.6},\quad n=2\ S=\frac{82}{3.4.5.6.7},$$

$$n=3\ S=\frac{860}{3.4.5.6.7.8},\ n=4\ S=\frac{9240}{3.4.5.6.7.8.9}$$

which can be easily verified.