$$=x\left(n+\frac{n(n-1)}{\lfloor 2}\cdot\frac{\frac{p}{q}-1}{n}\right)$$

$$\therefore\ x=\frac{2\left(\frac{p}{q}-1\right)}{n+1+(n-1)\frac{p}{q}}.$$

Hence $\sqrt[n]{\frac{p}{q}}=1+x=\frac{\overline{n+1}\,p+\overline{n-1}\,q}{\overline{n+1}\,q+\overline{n-1}\,p}.$

Second part follows at once, $\because x<\frac{1}{10^r}$ and 1st term neglected is $\frac{n(n-1)}{\lfloor 2}x^2$.

IX. From first identity

$(z-\frac{1}{xyz}-\frac{b}{xy}+a)(y-x)=0$ $\because$ x, y, z are unequal.

$$z-\frac{1}{xyz}-\frac{b}{xy}+a=0 \quad (1)$$

similarly $x-\frac{1}{xyz}-\frac{b}{yz}+a=0 \quad (2)$

" $y-\frac{1}{xyz}-\frac{b}{zx}+a=0 \quad (3)$

From (1) and (2) $b=xyz$,

$$\therefore\ a=\frac{1}{xyz}$$

Substituting which values each member of these $=us=0$.

10. Each of given fractions

$$=\frac{a(x+y+z)-b(x+y+z)}{x+y+z}$$

$\therefore\ ax-by=ay-bz=az-bx=az-bz=ax-bx$ $=ay-by$, whence $x=y=z$.

11. $(2+1)^n-(2-1)^n=2(n.2^{n-1}$

$$+\frac{n(n-1)(n-2)}{\lfloor 3}.2^{n-3}+\ldots)$$

$$\therefore\ 3^n=1^n+n.2^n+\frac{n(n-1)(n-2)}{\lfloor 3}2^{n-2}+\ldots$$

12. Since $\sin C=\sin(\pi-\overline{A+B})$ $=\sin(A+B)$.

$\therefore$ given expression becomes

$$\sin\overline{A-B}\sin\overline{A+B}+\ldots+\ldots$$

or $\sin^2 A-\sin^2 B+\sin^2 B-\sin^2 C+\sin^2 C$ $-\sin^2 A=0$.

13. We have $\sec l=\frac{a+b}{2\sqrt{ab}}$, etc. $=$ etc.

Proposed identity follows immediately.

14. Let $x=$ distance from O. to C.

$y=A$'s rate per hour.

$z=B$'s rate per hour.

$$\frac{\frac{x}{2}}{y}+\frac{\frac{x}{2}}{\frac{6y}{5}}=\frac{99}{40} \quad (1)$$

$$\therefore\ x=\frac{27}{10}y$$

$$y+\frac{y}{5}=z-\frac{z}{9}$$

$$\therefore\ z=\frac{27}{20}y$$

$$\frac{\frac{27}{20}y-1\frac{1}{2}}{y}=\frac{\frac{27}{20}y}{z}$$

$$\therefore\ y=\frac{20}{6}$$

$$\therefore\ x=9$$

$\therefore$ 9 mls. dist. from O. to C.

15. The pt. O where circle passing through B, C, touches AO is pt. where BC subtends the greatest angle, *i.e.* where BC appears longest.

$$\tan COB=\tan(AOC-AOB)$$

$$=\frac{\tan AOC-\tan AOB}{1+\tan AOC.\tan AOB}.$$

$$=\frac{\frac{AC-AB}{AO}}{\frac{AO^2+AC.AB}{AO^2}}$$

$$=\frac{BC}{2AO},$$

$$\because AO^2=AC.AB.$$

16. Let P be the object, PD its height $=x$ suppose.

Let $CD=z$.

$$\tan m=\frac{x}{a+b+z} \quad (1)$$

$$\tan 2m=\frac{x}{b+z} \quad (2)$$

$$\tan 3m=\frac{x}{z} \quad (3)$$

$$\therefore\ x=(a+b+z)\tan m \quad (4)$$

$$=(b+z)\frac{2\tan m}{1-\tan^2 m} \quad (5)$$

$$=z\frac{3\tan m-\tan^3 m}{1-3\tan^2 m} \quad (6)$$