3. What is the locus of the poles of all straight lines which pass through a given point?

4. If four points form a harmonic range, their polars with respect to any circle form a harmonic pencil. [For the polars all pass through the pole of the line on which the range lies; and the straight lines joining the four points to the centre are inclined at the same angles as the polars of the points.] Prove converse.

5. If AB be any chord of a circle, and P, Q be harmonic conjugates with respect to A, B, then the polar of P passes through Q, and the polar of Q passes through P. [Follows at once from Prop. 30.]

6. A, B, C, D **are four points taken in order on the circumference of a circle.** AD, BC **intersect at** P; AC, BD **at** Q; **and** AB, CD **at** R. **Show that the triangle** PQR **is such that each vertex is the pole of the opposite side.** [Let BD meet PR in T. Then B, Q, D, T form a harmonic range, and RP, RC, RQ, RB a harmonic pencil. Hence if RQ cut AD in X and BC in Y, then P, D, X, A form a harmonic range, and also P, C, Y, B. But the polar of P cuts AD and BC harmonically. Therefore QR is the polar of P. Similarly QP is the polar of R; a. . . refore PR is the polar of Q.]

The triangle PQR, each of whose sides is the polar of the opposite vertex, is said to be **self-conjugate** with respect to the circle.

7. Employ the preceding to draw tangents to a circle from a given point, using a ruler only, the centre of the circle not being known.

8. P, Q are any two points in the plane of a circle whose centre is C. PX is the perpendicular on the polar of Q, and QY the perpendicular on the polar of P. Show that PC . QY = QC . PX. [If PX, QY meet in R, then PCQR is a parallelogram. Draw perpendiculars CA, RB on PX, PC. Then PA . PR = PB . PC. Also CQ . CN = CP . CM, if CQ intersect (Q) in N, and CP intersect (P) in M; etc.]

9. If two circles cut orthogonally, and AB be any diameter of one of them, the polar of A with respect to the other circle passes through B.