These preliminary investigations being made, we are now prepared to establish the general equation of the three moments.

In Fig. 5, let A, B and C be three consecutive points of support of a continuous beam, the spans considered being, as shown, l_a and l_c. R_a, R_b and R_c are the reactions at A, B and C; and S_a', S_a, S_b', S_b, S_c', S_c are those portions of R_a, R_b and R_c caused respectively by the loads on the span to the left and those on the span to the right; this can be understood by examining the diagram. Let us call the moments at A, B and C respectively M_a, M_b and M_c, and let us take the origin of coordinates for the span l_a at A and for the span l_c at C.

Referring to Eq. 5, the value of D will reduce to zero if the points considered be two points of support, for they are upon the same level; we can then write

$$0 = \Sigma_b^a \frac{x\,M}{E\,I}\,dx,$$ the Σ_b^a denoting that the points considered are A and B.

As E and I are both finite quantities, we must have $0 = \Sigma_b^a x\,M\,dx$. Substituting the value of M from Eq. 8, gives:

$$0 = \Sigma_b^a \left[x\,M_1\,dx + M_a \frac{(l_a - x)\,x}{l_a}\,dx + M_b \frac{x^2}{l_a}\,dx \right]$$

or writing the integral sign instead of Σ

$$0 = \int_0^{l_a} \left[x\,M_1\,dx + M_a\,x\,dx - M_a \frac{x^2}{l_a} dx + M_b \frac{x^2}{l_a} dx \right]$$

$$0 = \int_0^{l_a} M_1\,x\,dx + M_a \frac{l_a^2}{2} - M_a \frac{l_a^2}{3} + M_b \frac{l_a^2}{3}$$

or multiplying by $\frac{6}{l_a}$

$$0 = M_a\,l_a + 2\,M_b\,l_a + \frac{6}{l_a} \int_0^{l_c} M_1\,x\,dx. \quad \text{[Eq. 9.]}$$

Passing to the span l_c, we can establish in a similar manner the following equation:—

$$0 = M_c\,l_c + 2\,M_b\,l_c + \frac{6}{l_c} \int_0^{l_a} M_2\,x\,dx. \quad \text{[Eq. 10.]}$$

Adding Eqs. 9 and 10, gives

$$0 = M_a\,l_a + 2\,M_b\,(l_a + l_c) + M_c\,l_c + \frac{6}{l_a} \int_0^{l_a} M_1\,x dx + \frac{6}{l_c} \int_0^{l_c} M_2\,x\,dx. \quad \text{[Eq. 11.]}$$

Let us suppose for an instant that there is only one weight P which produces the infinite number of moments represented by M_1 in the expression $\int_0^{l_a} M_1\,x dx$,

The reaction at A will be $P \frac{l_a - z}{l_a}$ and the value of M_1 for any point between A and the point of application of P will be $P \frac{l_a - z}{l_a} x$: the reaction at B will $P \frac{z}{l_a}$ and the value of M_1 for any point between B and the point of application of P will be $P \frac{z}{l_a} (l_a - x)$;

whence we can write the equation

$$\int_0^{l_a} M_1\,x\,dx = \int_0^z P \frac{l_a - z}{l_a} x^2\,dx + \int_z^{l_a} P \frac{z}{l_a} (l_a - x)\,x\,dx$$

$$= P \frac{l_a - z}{2} \frac{z^3}{3} + Pz \left(\frac{l_a^2}{2} - \frac{z^2}{2} \right) - P \frac{z}{l_a} \left(\frac{l_a^3}{3} - \frac{z^3}{3} \right)$$

$$= \frac{1}{6} P\,z\,(l_a^2 - z^2)$$

and for any number of loads

$$\int_0^{l_a} M_1\,x\,dx = \frac{1}{6} \Sigma^a P\,z\,(l_a^2 - z^2).$$

Similarly $\int_0^{l_c} M_2\,x\,dx = \frac{1}{6} \Sigma^c Pz\,(l_c^2 - z^2)$

Substituting these values in Eq. 11, gives

$$0 = M_a\,l_a + 2\,M_b\,(l_a + l_b) + M_c\,l_c + \frac{1}{l_c} \Sigma^a P\,z\,(l_a^2 - z^2) + \frac{1}{l_c} \Sigma^c P\,z\,(l_c^2 - z^2)$$

which is the ordinary form of the equation of the three moments, Σ^a and Σ^c denoting that the summations are each to extend over one span from A and C respectively.

Taking moments about the right hand end of l_a, gives $S_a\,l_a - \Sigma^a P\,l_a - z + M_a = M_b$ from which we have

$$S_a = \Sigma^a P \frac{l_a - z}{l_a} - \frac{M_a - M_b}{l_a}$$

Taking moments about the left hand end of the same span, there results

$S_b'\,l_a - \Sigma^a Pz + M_b = M_a$ from which we have

$$S_b' = \Sigma_a P \frac{z}{l_c} - \frac{M_a - M_b}{l_a}$$

$$\text{Similarly } S_b = \Sigma^c P \frac{z}{l_a} + \frac{M_c - M_b}{l_c}$$

$$\text{and } S_c' = \Sigma^c P \frac{l_c - z}{l_c} - \frac{M_c - M_b}{l_c}$$

By inspecting Fig. 5, as before stated, we can see that

$$R_a = S_a' + S_a$$
$$R_b = S_b' + S_b$$
$$R_c = S_c' + S_c$$