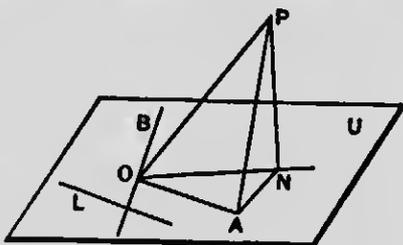


*Def. 3.* The angle between two non-complanar lines is the angle between two intersecting lines respectively parallel to the given lines.

**12. Theorem.** The angle between a line and its projection on a plane is less than the angle between the given line and any planar line not parallel to the projection.

The line  $PO$  meets the plane  $U$  in  $O$ ;  $ON$  is the projection of  $OP$  on  $U$ ;  $OA$  is a line through  $O$ , parallel to the planar line  $L$ , which is not parallel to the projection  $ON$ .



Then  $\angle PON$  is  $< \angle POA$ .

*Proof.* From  $P$  draw  $PN$  perpendicular to  $ON$ .  $PN$  is normal to the plane  $U$  (Art. 11. Def. 1).

Take  $OA = ON$  and join  $PA$  and  $AN$ .

Since  $\angle PNA = 90^\circ$ ,  $PA$  is  $> PN$ .

And in the triangles  $POA$  and  $PON$ ,  $PO$  is common,

$OA = ON$ , and  $PA > PN$ ;

$\therefore \angle POA$  is  $> \angle PON$ . (P. Art. 67.)

And as  $L$  is any planar line not parallel to  $ON$ , the  $\angle PON$ , between  $PO$  and its projection on  $U$ , is less than that between  $PO$  and any line in the plane, not parallel to  $ON$ .

*Cor. 1.* Since two intersecting lines make with one another two angles which are supplementary (P. Art.