VELOCITY FORMULAS

Their History and Investigation of Their Relative Accuracy—Report Prepared for the Miami Conservancy District

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As an element in estimating the discharge of a stream, the velocity of the water was first introduced by Castelli in 1628. In 1643 Torricelli discovered that the velocity of water flowing freely from a small orifice, is equal to the velocity of a body falling in a vacuum a distance equal to the depth of the orifice below the water surface.

Guglielmini, whose works appeared near the end of the 17th century, adopted the theories of Torricelli and proposed the celebrated parabolic theory of river velocity which may be briefly stated as follows: Any particle x feet below the surface of a stream will tend to move at the same velocity that it would if issuing from an orifice x feet below the surface of a reservoir. Although this theory would indicate that the velocity at the surface of a stream is zero, and that the maximum velocity occurs at the bottom, it was adopted by many eminent scientists and was not disproven until Pitot published the results of his experiments in 1730-1738, his experiments consisting of measurements of velocities at different depths by the aid of the tube which bears his name.

Chezy the Real Beginner

In 1738 Daniel Bernoulli published his noted works on hydraulics in which he proposed the well-known Bernoulli theorem. In 1753 Brahms observed that the velocity does not accelerate in accordance with the law of gravity, but that it tends to assume a constant value. He pointed out the friction of the water against the bed and sides of the channel as the force opposing the acceleration and assumed that the resistance is proportional to the hydraulic radius. In 1775 Chezy put the theories of Brahms into algebraic form, introducing the well-known Chezy formula. Although Varignon, in 1725, reduced the theories of Guglielmini to algebraic equations, the work of Chezy marks the real beginning of velocity formula studies.

Dubuat, whose work was published in 1786, started with the law that when water flows uniformly the forces which keep it in motion are equal to the sum of all resistances. He reasoned that the best method to deduce a formula is to find by experiment algebraic expressions for these two opposing forces and then equate them. Following these ideas he made a number of experiments upon pipes and small channels and from them deduced a formula for velocity. He established the principles that the motive force of the water is due entirely to the surface slope, that the resistances are due to viscosity and friction on the sides and bed of the channel, and that the resistance is independent of the weight or pressure of the water.

Coulomb published a paper in 1800 in which he discussed the laws of friction between fluids and solids. He showed that the resistances may be represented by a function consisting of two terms, one containing the first power of the velocity and the other, the second. Girard, in 1803, applied this theory of Coulomb's to the flow of water in open channels and deduced a formula which was more simple than Dubuat's.

De Prony's Contribution

In 1804, de Prony, by a discussion of experiments, corroborated the general conclusion of Coulomb's regarding the resistances, but showed that the two terms should be modified by independent co-efficients instead of by a common one as had been proposed by both Coulomb and Girard. He discussed the measurements of Dubuat and others, and from them deduced values of the coefficients for pipes and canals.

In 1814 Eytelwein, from a study of 91 observations on rivers and canals covering a wide range of conditions, proposed new values for the coefficients in de Prony's formula. De Prony's formula with Eytelwein's coefficients was used extensively for several years.

Additional formulas were proposed by Young in 1808, by Berlanger in 1828, by Lombardini in 1844, by Taylor in 1851, by Ellet in 1851, and by Stevenson in 1858.

In 1851, de Saint Venant proposed the first of the formulas which have been termed exponential formulas; that is, he proposed a formula based on the assumption that the velocity does not vary as the square root of the slope times the hydraulic radius as was assumed by Brahms, Chezy and others, but that it varies according to some fractional power of the slope times some other fractional power of the hydraulic radius, both terms being multiplied by a coefficient, as in the Chezy formula.

Formulas of similar nature were proposed by Lampe, Flamant and Hagen. The last, however, concluded that the exponents of the slope and the hydraulic radius are not constants except for given classes of pipes or channels.

During the latter half of the 19th century, numerous formulas have been proposed. A few of these, such as Kutter's and Bazin's, have been based on long and careful studies by able investigators. Others have been based on but poor foundations.

German Formulas Lack Merit

The studies and comparisons of the various German velocity formulas, which have been developed on the assumption that a roughness coefficient is not necessary, show that no one of them possesses sufficient merit to warrant its use. In fact, no one of them could be considered to be definitely better than the others. Although the results determined by the Hessle equation show up fairly well in certain instances, they are among the worst in others. The velocities calculated by the Siedek equation are apparently the most erratic and discrepant of all, being as much as 100 per cent. in error in certain cases. The final comparison of the formulas shows conclusively that in any general formula for computing velocities in open channels, a roughness coefficient is necessary.

The exponential formulas thus far proposed do not possess any important advantages. The equation recently proposed by Barnes is not as accurate as his comparisons indicate. In fact, the type of formula that he assumes—namely, one in which C' (the coefficient of roughness in exponential formulas), is a constant for a given class of roughness, does not appear to be feasible. If any exponential equation were to be recommended for general use it would be one similar to that proposed by Williams.

The claim that C' is less variable than C (the coefficient of roughness in Chezy's formula), does not seem to be correct for natural river channels. In fact, for such conditions the reverse appears to be true. Whatever constancy is gained by the higher exponent of R (the hydraulic radius of the cross-section of channel), is evidently more than offset by some other factor or factors in favor of the Chezy equation. This shows that no saving in the amount of engineering judgment required would result from the adoption of an exponential formula.

Biel Inferior to Bazin

The Biel velocity formula, which has the distinction of containing a temperature correction, is inferior to Bazin's formula. In several series of experiments in which the roughness conditions were constant, the roughness factor in the Biel formula was found to vary more than the corresponding coefficient in the Bazin equation. For open channels the temperature term is negligible in all but very extreme cases. Even in those instances in which it is appreciable, it is, of course, a question as to whether it should be considered. There is no data at hand to show that the effect assumed by Biel is correct. The publication in which Biel proposed his formula is not available. It is said, however, that he recommended his formula for the computation of velocities of gases and liquids in general, instead of for water only. In view of this fact it seems probable that he introduced the temperature term on account of the effect of temperature on the flow of gases and such liquids as oils rather than on account of its effect on the flow of water.