Equating coefficients,

$$a^2 = 1 \quad \therefore \ a = 1$$

$$2ab = 1 \quad \therefore \ b = \tfrac{1}{2}$$

$$2ac + b^2 = 1 \quad \therefore \ c = \frac{1 - b^2}{2a} = \frac{3}{8}$$

$$2ad + 2bc = 0 \quad \therefore \ d = -\frac{bc}{a} = -\frac{3}{16}$$

$$\therefore \ \sqrt{1 + x + x^2} = 1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2 - \frac{3}{16}x^4 \ldots.$$

Ex. 173. What relation must exist among the quantities p, q, r, s in order that x^2+px+q and x^2+rx+s may have a common factor.

Let the common factor be $x+a$, then the expressions may be written

$$(x+a)(x+\frac{q}{a}), \text{ and } (x+a)(x+\frac{s}{a}),$$

since the last terms in the products will evidently be q and s as they should be.

Then we must have,

$$a + \frac{q}{a} = p, \qquad a + \frac{s}{a} = r.$$

$$\therefore \ a^2 - ap = -q, \qquad a^2 - ar = -s.$$

And eliminating a^2 and a by determinants.

$$a = \frac{\begin{vmatrix} q & 1 \\ s & 1 \end{vmatrix}}{\begin{vmatrix} p & 1 \\ r & 1 \end{vmatrix}} = \frac{q-s}{p-r}$$

$$\text{and } a^2 = \frac{\begin{vmatrix} p & q \\ r & s \end{vmatrix}}{\begin{vmatrix} 1 & p \\ 1 & r \end{vmatrix}} = \frac{qr - ps}{p-r}.$$

$$\therefore \ (p-r)(qr-ps) = (q-s)^2,$$

is the necessary relation.