

just as, in the interrogation of Nature, it is necessary not only to observe, but to devise ways for observation, so, in the development of the Sciences of Number and Space, it is necessary not only to follow out the consequences prescribed by laws of reason, but to invent ways of bringing those laws into operation. In Geometry this is needed very early. Take Euclid's thirty-second proposition—which ought to be very early. Something has to be done, some way devised of bringing our previous knowledge of the properties of parallels to bear upon the facts of a triangle; and a characteristic exercise of invention, and the imagination which invention presupposes, is involved in success. If finding out direct consequences *per se* delight some minds, finding out how to find them out delights the rest. The pleasure to be derived from inventing geometrical methods, and discovering geometrical truths, is indeed enormous, and such as should make this science the most attractive in the curriculum.

When our pupils have reached this stage of being able to take pleasure in the logical development of Mathematics, and delight in their own powers of invention and discovery, they may tell us that they have not much ability for it, but they will hardly tell us that they have no taste; and if they leave school at this point they will leave it with some safeguard, in acquired capacity, against the jumping to conclusions, and treacherous reasoning, and uncertain credulousness, and equally uncertain incredulousness, that are only too common. In this dry clear atmosphere of absolute certain truth and unemotional thought, they have learned to think precisely and impartially, and have that power to carry with them into the much more difficult arena of actual life. Moreover, they have acquired a habit of looking closely into the *rationale* of all things—of getting to the bottom of a subject. They will not be put off with insufficient reasons; an indissoluble association between statement and proof has been wrought in their minds. We all know the story of the Senior Wrangler who asked of Paradise Lost, "What does it prove?" but it would be still more like a Senior Wrangler to ask, "How is it proved?" These two questions are indeed typical of the double training which mathematical study gives, accustoming us to look back for the reasons and forward for the consequences at once. Besides, our pupils will have acquired a certain ingenuity of invention, a power of concentrating attention, and a habit of expressing ideas clearly. These are valuable faculties in understanding oneself and the world, and the last is bound up with mathematical thought in an intimacy that cannot be too strongly insisted on. Mathematics is nothing unless it is clearly expressed; there is no escape from the necessity, and the result is an advance in the faculty of expression, more remarkable, it seems to me, than any that the study of languages can secure. The advance is indeed different in kind. Language study enriches our language; exact science gives us the command of it, requiring us to use it with the most precise sense of its meaning. For myself, I believe that one year's study of Mathematics gave me a greater power over language than many previous years of English reading and French and German study.

Nor is this all. The product in mental training of mathematical study is more than these invaluable qualities of hard-headedness, as above described. These are the result of its methods. The result of its subject matter is to be found in the remarkable development of the imagination which its study produces. The popular type of the mathematician is the mere algebraist, who does not see, or dream of seeing, that there is, as

the greater men declare, at bottom of every algebraical conception a geometrical foundation. He does not call upon his imagination, because he is content to arrive at his results by accurate numerical reasoning, and does not want to picture them in terms of space imagination as well. The true mathematician is a different kind of person from this: he seeks for a form under all his thoughts; he thinks in terms of form; he sees the details of all form around him; he makes the most elaborate space-pictures in his mind at will—his imagination is the most remarkable thing about him. As a consequence, he is the most enthusiastic admirer of natural scenery, and remembers what he has seen with marvellous accuracy. It is inevitable, indeed, that he should be a passionate lover of beauties of form, even if of poetical appreciation he did not possess one iota. Geometry is the most perfect training of the physical imagination, and, as such, subserves the ends of æsthetic development, and all other ends that imagination forwards. As another matter of fact, the two geometrical nations *par excellence*, Greece and France, have built the most beautiful cities in the Ancient and Modern worlds respectively. We are not a geometrical nation, and no one would think, indeed, from our Cambridge text-books, that Mathematics is so pre-eminently elegant as it is thought and felt to be in France, or that it is, as Gauss says, a science of the eye. Our mathematical faculty lies in our great industry, and the positive pleasure we find in doing hard things in the hardest possible way,—the hardest possible way being all very well for the strong-brained mathematicians who write text books at Cambridge, whereas the most elegant possible way has æsthetic and other educational advantages which it might be well for the youth of the country if these strong-brained personages could come to see.

Professor Sylvester has a word bearing on this subject of the educational value of Mathematics in general, and Geometry in particular:—

"Some people have been found to regard all mathematics, after the 47th proposition of Euclid, as a sort of morbid secretion, to be compared only with the pearl said to be generated in the diseased oyster, or, as I have heard it described, 'une excroissance malade de l'esprit humain.' Others find its justification, its 'raison d'être,' in its being either the torch-bearer leading the way, or the handmaiden holding up the train of Physical Science; and a very clever writer, in a recent magazine article, expresses his doubts whether it is, in itself, a more serious pursuit, or more worthy of interesting an intellectual human being, than the study of chess problems or Chinese puzzles. What is it to us, they say, if the three angles of a triangle are equal to two right angles, or if every even number is, or may be, the sum of two primes, or if every equation of an old degree must have a real root. How dull, stale, flat, and unprofitable are such and such like announcements! Much more interesting to read an account of a marriage in high life, or the details of an international boat-race. But this is like judging of Architecture from being shown some of the brick and mortar, or even a quarried stone of a public building, or of painting from the colours mixed on the palette, or of music by listening to the thin and screechy sounds produced by a bow passed haphazard over the strings of a violin. The world of ideas which it discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connexion of its parts, the infinite hierarchy and absolute evidence of the truths with which it is concerned,—these, and such like, are the surest grounds of the title of Mathematics to human regard, and would