N. B.—It is necessary to find by trial whether any or all of these roots apply to the given equation. They may belong only to the conjunct equation introduced by squaring or some to one and some to the other. See *Handbook*, p. 260.

(*b*) Given $x^2+(x^4y^2)^{\frac{1}{3}}=208$

and $y^2+(x^2y^4)^{\frac{1}{3}}=1053$. Put $y=vx$.

$\therefore\ x^2+(x^6v^2)^{\frac{1}{3}}=208=x^2(1+v^{\frac{2}{3}})$

and $v^2x^2+(x^6v^4)^{\frac{1}{3}}=1053=x^2(v^2\times v^{\frac{4}{3}})$

$\therefore\ \dfrac{v^{\frac{4}{3}}(1+v^{\frac{2}{3}})}{1+v^{\frac{2}{3}}}=\dfrac{1053}{208}=v^{\frac{4}{3}}=\dfrac{81}{16}\quad\therefore\ v=\dfrac{27}{8}.$

$\therefore\ x^2=208\div(1+v^{\frac{2}{3}})=64\quad\therefore\ x=\pm 8$

Substituting, &c., on the whole we get

$x=8,\ -4,$ or $8(19\pm 8\sqrt{6})$

$y=4,\ 1,$ or $8(5\pm 2\sqrt{6})$.

6. (*a*) Book work.

(*b*) $(a+b-c)(a+c-b)=a^2-(b-c)^2,\ \therefore\ <a^2$

$(b+a-c)(b+c-a)=b^2-(a-c)^2,\ \therefore\ <b^2$

$(c+a-b)(c+b-a)=c^2-(a-b)^2,\ \therefore\ <c^2$

$\therefore\ (a+b-c)^2.\ (b+c-a)^2.\ (c+a-b)^2<a^2b^2c^2$

or $abc>(a+b-c)(b+c-a)(c+a-b)$, which is the first part.

2ND PART. $a^2+b^2>2ab$, (A) $\therefore\ a^3+ab^2>2a^2b$

and also, $b^3+a^2b>2ab^2$,

$\therefore a^3+b^3>a^2b+ab^2$

and by symmetry $a^3+c^3>a^2c+ac^2$

$b^3+c^3>b^2c+bc^2$

whence by addition,

$a^3+b^3+c^3>\frac{1}{2}\ (a^2b+b^2c+c^2a+\&c.)$ B.

But $a^2c+b^2c>2abc$ from (A)

$ab^2+ac^2>2abc$

$bc^2+a^2b>2abc$, $\therefore$ by addition

$(a^2b+b^2c+c^2a+\&c.)>6abc$ (C)

$\therefore$ from (B), $a^3+b^3+c^3>3abc$

But $(a+b+c)^3=(a^3+b^3+c^3)+3(a^2b+b^2c+\&c.)+6abc$

$\therefore (a+b+c)^3\ >\ 3abc\ +\ 18abc\ +\ 6abc$

or (" $)^3>27abc$

$\therefore \left(\dfrac{a+b+c}{3}\right)>abc.$

N.B.—The theorem is not true when a, b, c are any +ve Nos. *whatever*. If $a=b=c$, the inequalities become equalities; a, b, c must be unequal.

7. Put $\dfrac{x^2+ax+b}{x^2+cx+d}=m$

$\therefore x^2+ax+b=mx^2+mcx+md$

$\therefore x^2(1-m)+x(a-mc)+(b-md)=o$

or $kx^2+rx+q=o$ if we write k for $(1-m)r$ $r=a-mc$, and carry $q=b-md$.

Now in order that x may possible r^2 must be $>4kq$, see text-books: or $(a-mc)^2>4(1-m)\ (b-md)$

i.e., $a^2-2amc+m^2c^2>4b-4md-4bm+4m^2d$

or, $m^2\ (c^2-4d)+m\ (4b+4d-2ac)+(a^2-4b)>o$

i.e., (A) $pm^2+sm+w>o$, if we put

$c^2-4d=p,\ 4b+4d-2c=s,$ and $a^2-4b=w$

$\therefore$ Hence all values of m lie between α and β, the equation carry $pm^2+sm+w=o$.

$\therefore$ α and β are the LIMITS of the possible values of the fraction. See Colenso, Pt. II., p. 206.

8. (*a*) Bookwork. Limit$=a\div(1-r)$.

(*b*) Ans. $=\frac{3}{4}\div(1-\frac{7}{8})=6$.

9. (*a*) Bookwork. $\underline{|n}\div\underline{|p}\,.\,\underline{|q}\,.\,\underline{|r}$.

(*b*) $n=8$, three *a*'s, three *m*'s

$\therefore$ Ans.$=\underline{|8}\div\underline{|3}\,.\,\underline{|3}=1120$.

10. $(1+x)^n=1+nx+\dfrac{n(n-1)}{\underline{|2}}x^2+\dfrac{n(n-1)(n-2)}{\underline{|3}}x^3+\&c.$

$\therefore\ (1-x)^n=1-nx+$ " $-$ " $+\&c.$

$\therefore\ (1-x^2)^n=1-nx^2+$ " x^4- " $x^6+\&c.$

$\therefore\ (1-x^2)^{-n}=1+nx^2+\dfrac{n(n+1)}{\underline{|2}}x^4+\dfrac{n(n+1)(n+2)}{\underline{|3}}x^6+\&c.$

$\therefore\ (1-x^2)^{-\frac{1}{2}}=1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6+\frac{35}{128}x^4+\&c.$

3rd part.

Put $\dfrac{2n+1}{2n-1}=x,\ \therefore\ \dfrac{1+x}{1-x}=-2n$

$\therefore\ -n=(1+x)\div 2(1-x)$

Also $\dfrac{2}{1-x}=-(2n-1)$

$\therefore\ n(2n-1)=\dfrac{1+x}{(1-x)^2}$, and by division, or by expansion this $=1+3x+5x^2+\&c.\ \ +x(2n-1)x^{n-1}$

Restore the value of x and the theorem is established.

11. Bookwork. 5th term=6th term=7⅞.

Practical Department.

A REMARKABLE MOSAIC.

LAUGHABLE ERRORS.

One pupil when asked to describe the "*Missouri Compromise*" said: "It is a muddy stream that flows into the Mississippi."

Another said: "An *assessor* is a man appointed by the government to *appropriate* the taxes." He builded wiser than he knew.

Still another said in reply to the question about "*Salem Witchcraft:*" Salem Witchcraft and Roger Williams were missionaries to the Indians.

These will do to go with the teachers who thought that Horace Greely commanded the Greely Expedition, that the Cotton Gin was a kind of whisky made of Cotton, that squatter sovereignt took its name from John Squatter, an early settler in Kansas. Bring in some more.—*The Moderator.*

THREE INCIDENTS.

I.

A teacher was examining the slates of a class of beginners in writing, after some dictation exercises. When nearly through, one whispered as her slate was being examined. "We are doing ever so much better; aren't we, Mrs. B——?"

"Why do you think so?"

"Because so far, you have hardly had to tell one that they forgot to begin the sentence with a capital, and end with a period. And you read them ever so much faster, too."

How very carefully a teacher is watched.

II.

Some girls of thirteen, when told of the writer's intention of visiting them at school, eagerly said: "Oh please don't, please don't come. You are not used to seeing scholars act as we do. You could not stay. We would not have you see us there for anything."

"Why do you act in this way? Do you study?" was asked.

"We did study at first, and behaved real well, but the teacher never said a word, and did not seem to care, and some were having fun all the time, so now we all act alike. There is no use of trying to study, or anything."

How many pupils in every school are discouraged and give up trying, because the teacher "does not seem to care?"

III.

A teacher was troubled by the overcrowded condition of his school-room. Appeals for additional seats were disregarded by the directors. One day, when all the available seating facilities were in use, and a boy was ensconced in the teacher's chair and a few more on the floor, he sent for his Board. Mr. A. came in, and was warmly received. He looked about somewhat hesitatingly, and said. "Well, Mr. A. I should be glad to give a chair if I had one, but I am just out. Make yourself at home; sit down on the stove." Mr. A., to the amusement of the pupils, did so—the weather being warm, there was no fire. Shortly after, director number two appeared. He was received with equal cordiality by the teacher, and, from necessity, took his position with number one. Number three put in an appearance a little later, and was offered a place by the side of his official colleagues. But about that time it began to draw upon the minds of the triumvirate that the teacher was less innocent than his "childlike and bland" countenance indicated. The president called him to one side with, "Mr. R., I am a little busy, and will call again. How many do you need?" It is needless to say that an adequate supply of desks, with all the modern improvements, were on hand in the shortest possible time.—*Teacher's Institute.*