(b) Assume
$wx(w+x)^2+wy(w+y)^2+wz(w+z)^2+yz(w^2+x^2)+zx(w^2+y^2)+xy(w^2+z^2)-2xyzw=0$

i.e. $wx(w+x)^2+wy(w+y)^2+wz(w+z)^2+yz(w^2+x^2)+zx(w^2+y^2)+xy(w^2+z^2)=2xyzw$

Put $w=0$ i.e. $x+y+z=0$ in given relation and we have $0+0+0+yz(x^2)+zx(y^2)+xy(x^2)$ on the left hand

i.e. $xyz(x+y+z)$ which must $=0$ since $x+y+z=0$

∴ w is a factor of the left hand member. Hence by symmetry $xyzw$ is a factor

i.e. left hand $=Nwxyz$ where N is some *numerical* factor, for being of only four dimensions the left hand side can have no more literal factors. To find N, put $w=1$, $x=2$, $y=-2$, $z=-1$.—[N.B. We must be careful to assume values which agree with the relation $w+x+y+z=0$.] We then have

$2(3)^2+(-2)(-1)^2+0-(2)(5)+(-4)(-2)=4N$

whence $8=4N$ and $N=2$ as required. Thus the given relation is true.

It will be useful to the reader to solve this example synthetically thus $w+x+y+z=0$ ∴ $(w+x)^2=(y+z)^2$; $(w+y)^2=(x+z)^2$; $(w+z)^2=(x+y)^2$.

Hence
$wxy(w+x+y+z)+wxz(w+x+y+z)+wyz(w+x+y+z)+xyz(w+x+y+z)=0$

i.e. $wx(y^2+z^2)+wy(x^2+z^2)+wz(x^2+y^2)+yz(w^2+x^2)+xz(w^2+y^2)+xy(w^2+z^2)+4wxyz=0$

i.e., $wx(y+z)^2+wy(x+z)^2+wz(x+y)^2+yz(w-x)^2+zx(w-y)^2+xy(w-z)^2+4wxyz=0$

∴ $wx(w+x)^2+wy(w+y)^2+wz(w+z)^2+yz(w-z)^2+zx(w-y)^2+xy(w-z)^2+4wxyz=0$ and this is the given relation.

3. (a) Multiply through by 30

$18x-12-5x+30+\frac{90x}{51}=x+\frac{3x}{17}$

$12x+18=\frac{3x}{17}$

$4x+6=\frac{x}{17}$

$67x=-102$ $x=-\frac{102}{67}$.

(b) From (2) $a(x+y)=ab-xy$

∴ $a^3\{(x^3+y^3)+3xy(x+y)\}=a^3b^3-x^3y^3-3abxy(ab-xy)$

And from (1) $a^2b^3+3xy(x+y)=a^3b^3-x^3y^3-3abxy(ab-xy)$

∴ $xy=0$ i.e. either $x=0$ or $y=0$. If the former $y^3=b^3$ or $y=b$, if the latter $x=b$.

(c) Write the equations

(1) $x+y+z=3$
(2) $x^2+y^2+z^2=3$
(3) $x^3+y^3+z^3=6$

Square (1) and substitute (2)
and $(xy+yz+zx)=3$, (4).

Cube (1) and substitute (3),
$(x^3+y^3+z^3)+3(x+y+z)(xy+yz+zx)-3xyz=27$

i.e., $3xyz=6$, $xyz=2$, $xy=\frac{2}{z}$ (5).

Substitute (5) and (1) in (4)
$xy+yz+zx=xy+z(x+y)=3$

∴ $\frac{2}{z}+z(3-z)=3$, a quadratic which gives two values for z, from which we may find corresponding values for x, and y.

4. $(n+2)^{th}$ term of $A.P=\frac{n+2}{2}\{b(n+1)-a(n-1)\}=\frac{n+2}{2}.x$, say (A)

" " $G.P=\frac{b^{n+1}}{a^n}$ (B)

" " $H.P=\frac{2ab}{(n+2)\{a(n+1)-b(n-1)\}}=\frac{2ab}{(n+2)y}$, say (C)

Now when A, B, C are in $G.P$ $\frac{A-B}{B-C}=\frac{A}{B}$; hence we must have

$$\frac{\frac{(n+2)x}{2}-\frac{b^{n+1}}{a^n}}{\frac{b^{n+1}}{a^n}-\frac{2ab}{(n+2)y}}=\frac{\frac{(n+2)x}{2}}{\frac{b^{n+1}}{a^n}}$$

i.e., $1-\frac{b^{n+1}}{a^n}\cdot\frac{2}{(n+2)x}=1-\frac{a^n}{b^{n+1}}\cdot\frac{2ab}{(n+2)y}$

or $\frac{b^{2n+2}}{a^{2n}}=\frac{abx}{y}$

∴ $\frac{b^{2n+2}}{a^{2n+2}}=\frac{bx}{ay}$, also $\frac{ab^{2n+1}}{ba^{2n+1}}=\frac{ax}{by}$. Subtracting 1 from each side

$\frac{b^{2n+2}-a^{2n+2}}{a^{2n+2}}=\frac{bx-ay}{ay}$, and $\frac{ab^{2n+1}-ba^{2n+1}}{ba^{2n+1}}=\frac{ax-by}{by}$

Dividing one equation by the other

$\frac{b^{2n+2}-a^{2n+2}}{ab(b^{2n}-a^{2n})}=\frac{bx-ay}{ax-by}=\frac{n+1}{n-1}$ when we restore the values of x and y and strike out the common factor (b^2-a^2).

UNIVERSITY OF TORONTO.

JUNIOR MATRICULATION, 1882.

MATHEMATICS.—PASS.

Examiner—F. Hayter, B.A.

1. The interest on a sum of money for two years is $349.58, and the discount on the same sum for the same time is $310.74; simple interest in both cases. Find the rate per cent., and the time.

2. A. in Toronto pays B. in Paris 1000 francs by a bill of exchange on London, exchange at Paris being 25·25 francs for £1 sterling. Find the amount of the bill, and its value in currency (£1=$4.86⅔). When the bill reaches Paris exchange is at 25·23. Find the amount in francs for which the bill sells.

3. Simplify

(i) $\frac{x^2-15x+54}{x^2-7x+10}\times\frac{x^2-5x}{x^2-2x-63}\times\frac{x^2+5x-14}{x^2-6x}$.

(ii) $\frac{2\sqrt{2}+\sqrt{3}-1}{\sqrt{3}+1}-\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}-\frac{2\sqrt{2}+\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$.

4. Divide
$a^{4n-1}-a^{2n-1}+2a^{2n-2}-a^{2n-3}$ by $a^{2n}+a^n-a^{n-1}$.

Divide by Horner's method
$x^8+5x^6+11x^4+19x^2-36$ by $x^4-2x^3+2x^2+2x-3$.

5. Find the L. C. M. of
$(4x^3-4ax^2)$, $(3x^2-9ax+6x^2)$, and $(2x^3-8a^2x)$.

6. If the minute hand of a clock be 4 inches long and the hour hand 3 inches, find the times between 4 and 5 o'clock when their ends are 5 inches apart.

7. Solve

(i) $\sqrt{1+x^2}+x=a$.

(ii) $\begin{cases}12x-13y=7\\144x^2-156xy+169y^2=4729.\end{cases}$

(iii) $\begin{cases}x+y=z\\x^2+y^2=29\\xy=6.\end{cases}$

8. The opposite sides and angles of a parallelogram are equal to one another.

The diagonals of a parallelogram bisect each other. The angle between the diagonals of a rhombus is a right angle.

9. Upon the same straight line, and upon the same side of it, there cannot be two similar segments of circles, not coinciding with one another.

Similar segments of circles upon equal straight lines are equal to one another.

SOLUTIONS.

1. Int.=amt. of disct. ∴ (349.58−310.74)=int. for two yrs. on 310.74=$38.84

∴ 19.42 is int. on 310.74, what is the int. on $100?

Ans. $=(19.42\times100)\div310.74=97100\div15587=$&c.

2. 1 franc$=£\frac{4}{101}=\frac{4}{101}\times4.86\frac{2}{3}$ dollars

∴ 1000 francs$=1000\times\frac{4}{101}$ £'s sterling =&c.

$=1000\times\frac{4}{101}\times4.86\frac{2}{3}$, dollars. "

Value$=1000\times\frac{4}{101}\times25.23$ francs. "

3. (1) Expn. $=\frac{x-5}{x-2}$.

(2) $\frac{(\sqrt{2}-1)+(\sqrt{2}+\sqrt{3})}{\sqrt{3}+1}\left(1-\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}\right)-\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$

$=\frac{(\sqrt{2}-1)+(\sqrt{2}+\sqrt{3})}{\sqrt{3}+1}\cdot\frac{\sqrt{3}+1}{\sqrt{2}+\sqrt{3}}-\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$

$=\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}+1-\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}=1.$