2. (1) Let $x = a+b+c+d+e$, then given expression

$$3\{(x-e)^2 + \text{etc.}, -(a^2+b^2+c^2+d^2+e^2)\}$$
$$=3\{5x^2 - 2x(a+b+c+d+e)\}$$

$=9x^2$, substituting x for $a+b+c+d+e$, and $3(a+b+c+d+e)$ is square root sought.

(2) 0.

3. Show that $(x-a)^{10} - x^5a^5 + (x^2-ax+a^2)^5$ is exactly divisible by $x^3 - 2ax^2 + 2a^2x - a^3$.

Find the factors of $(a^2-b^2)^5 + (b^2-c^2)^5 + (c^2-a^2)^5$.

3. Divisor $=(x-a)(x^2-ax+a^2)$, the expression $(x-a)^{10} - x^5a^5 + (x^2-ax+a^2)^5$ vanishes when $x=a$, and is therefore divisible as stated.

Writing $x=a^2-b^2$, and $y=b^2-c^2$, and $\therefore x+y=a^2-c^2$ the given expression becomes $x^5+y^5-(x+y)^5$.

$$=x^5+y^5-(x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5).$$
$$=-\{5xy(x^3+y^3)+10x^2y^2(x+y)\}$$
$$=-5xy(x+y)\{x^2-xy+y^2+2xy\}$$
$$=-5xy(x+y)(x^2+xy+y^2).$$

$\therefore$ re-substituting for x, y and $x+y$, given expression $=-5(a^2-b^2)(b^2-c^2)$

$$(a^2-c^2)\{(a^2-b^2)^2+(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2\}$$
$$=5(a^2-b^2)(b^2-c^2)(c^2-a^2)\{a^4+b^4+c^4-a^2b^2-b^2c^2-c^2a^2\}$$

4. Show how to extract the square root of a quantity of the form $a+b\sqrt{-1}$.

(1) Find the square root of $-3-\sqrt{-16}$.

(2) Show that one of the fourth roots of -64 is $2(1+\sqrt{-1})$.

4. Bookwork.

(1) Given expression $=-(3+4\sqrt{-1})$, extracting square root of $3+4\sqrt{-1}$ in ordinary way we find it to be $(2+\sqrt{-1})$, and square root required is $-1+2\sqrt{-1}$.

(2) Required $2\sqrt{2}(-1)^{\frac{1}{4}} = R$, say in $x^4+1=0$, put $x+\frac{1}{x}=z$ and $z^2-2=0$, thus $z=\pm\sqrt{2}$, therefore $(x^2+x\sqrt{2}+1)(x^2-x\sqrt{2}+1)=0$, one of the roots of $x^2-x\sqrt{2}+1=0$, is $\frac{1+\sqrt{-1}}{\sqrt{2}}$ and $R=2(1+\sqrt{-1})$.

5. Solve the equations $ax+by=c$, $a'x+b'y=c'$.

Interpret the result when $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$.

5. Bookwork. See Todhunter's Algebra (larger) where, in chapter on simultaneous equations, the whole subject is discussed.

6. Solve the equations—

(1) $\frac{ax+b}{a+bx}+\frac{cx+d}{c+dx}=\frac{ax-b}{a-bx}+\frac{cx-d}{c-dx}$.

(2) $\frac{x}{b+c}+\frac{y}{c-a}=a+b$.

$$\frac{y}{c+a}+\frac{z}{a-b}=b+c.$$
$$\frac{z}{a+b}+\frac{x}{b-c}=c+a.$$

6 $\frac{ax+b}{a+bx}-\frac{ax-b}{a-bx}=\frac{cx-d}{c-dx}-\frac{cx+d}{c+dx}$

$$\frac{2ab(1-x^2)}{a^2-b^2x^2}=\frac{2cd(x^2-1)}{c^2-d^2x^2}$$

$\therefore$ $1-x^2=0$, and $x=\pm1$.

$$\text{or } \frac{ab}{a^2-b^2x^2}=\frac{cd}{c^2-d^2x^2}$$
$$abc^2-abd^2x^2=b^2cdx^2-a^2cd$$
$$ac^2(a+b)=bdx^2(ad+bc)$$
$$\text{and } x^2=\frac{ac^2(a+b)}{bd(ad+bc)}$$
$$x=\pm c\sqrt{\frac{a(a+b)}{bd(ad+bc)}}$$

or $x=\pm1$.

(2) By inspection we see at once that the roots are

$$x=b^2-c^2.$$
$$y=c^2-a^2.$$
$$z=a^2-b^2.$$

These may also be obtained by cross multiplication.

7. Find the relation between the roots and co-efficients of the equation $x^2+px+q=0$.

If the difference of the roots of the equation $x^2-(m-a)x+b^2$ is equal to the difference of the roots of the equation $x^2+(m-b)x+a^2$, show that $2m=5(a+b)$.

7. If α and β be the roots of the equation $x^2+px+q=0$, then $\alpha-\beta=\pm\sqrt{p^2-4q}$ making the necessary substitutions the result given, viz.: $2m+5(a+b)$ follows at once.