(c.) Tiberius Gracchus proposed an agrarian law, which proposed limiting the quantity of land held by individuals, and dividing the surplus land among the poor. When Octavius pronounced the veto, Tiberius secured a vote of the Tribes, expelling him from the Tribuneship.

(7.) Philip spent the early part of his life in Thebes, where he was detained as a hostage. While there, he studied Greek literature and politics, and when he returned to Macedon in 359 B.C., as its king, he organized a large army, which soon became a weapon of victory. His first steps were the seizure of Amphipolis, and the establishment of a military station at Philippi. Seizing his opportunity while the Athenians were engaged in a Social War, he interfered in the sacred war that arose between Thebes and Phocis. A victory over the Phocians left him master of Thessaly. He then overran Phocis, and gained a seat in the Amphyctionic Council. By laying siege to Perinthus and Byzantium, he first came into conflict with the Athenians. He was forced to raise the sieges of those cities, but a great defeat of the Athenians at Elatea brought about an alliance between Athens and Thebes. The allied forces suffered a terrible defeat at Cheronea, and Athens gladly accepted the humiliating terms of peace offered by Philip. Just two years after, in the noon of his glory, he was slain by an assassin during the procession of a marriage feast, 336 B.C.

8. Henry IV., first of the Bourbons, ascended the throne in 1589. Before he could consider his crown secure, he had to destroy the Holy League. This he did effectually, by the victories of Arques and Ivry (1590). In order to gain over the Romanists, he recanted his Protestantism. In 1598, however, he published the Edict of Nantes, granting to the Huguenots liberty of religion, and right to hold office. Sully was his chief adviser during most of the reign. The latter part of his reign was devoted to reforms in taxation and general government. In 1610 he was murdered by an assassin, who stabbed him through the window of his carriage, as he was setting off to head an army on the Rhine.

ANNA LIVING.

III. Mathematical Dept. & Correspondence.

To the Editor of the Journal of Education.

SIMCOE, January 2nd, 1873.

SIR,—Observing in the November issue of your *Journal* three solutions of a problem, which, it would seem, Mr. Cameron got inserted in the *Journal* for April, 1872. By some mistake either of the Department, or the Post-office of this place, I never got April's issue of the *Journal*, consequently, I cannot enter into the merits of the problem, beyond the ex-parte view given of it by Mr. Ryerson and the Mathematical Editor of the *Journal*. I will, however, say, from my knowledge of Mr. Cameron, being a fellow-contributor of mine to the Mathematical Department of a London periodical, that he invariably solved his problems upon correct mathematical principles. How he obtained the $21\frac{3}{7}$ per cent. I cannot divine, but with the $10\frac{10}{11}$ per cent., as given "by a majority of the commercial men of a western town," I agree, as being in accordance with the rule called "Equation of Payments," that is, viewing the problem as one belonging to simple interest, to which no well-trained mathematician would for a moment assent that it belonged.

$$\text{Equated time} = \frac{160(1+2+3+4+5 \text{———} 10)}{1600} = 5\tfrac{1}{2} \text{ years.}$$

$$r = \frac{600}{1000 \times 5\frac{1}{2}} = .10\tfrac{10}{11}$$

But "*Equation of Payments*" is founded upon "the supposition that what is gained by keeping certain payments after they become due, is equal to what is lost by paying other payments before they become due. This, however, is not exactly true: for the gain is the interest, while the loss is equal to the discount." In other words, when we solve problems according to this rule, which is by no means a correct one, we take into consideration *interest* as counteracting "*discount*," will the mathematical editor then be kind enough to indicate where he has obtained the "*text book principle*," which informs him that he is to subtract interest afterwards, when by the employment of this rule he supposes it to be expunged by the discount. Besides, from the very nature of the problem, which is drawn from business transactions taking place every day in the office of "The Building Association," the equated time for the ten annual payments of 160 dollars each, and to discharge a debt not of a thousand dollars but of sixteen hundred dollars, is five and a half years.

The mortgage given as collateral security for the debt, states that for one thousand dollars of current funds, well and duly paid, the mortgagor agrees to pay one thousand six hundred and forty-dollars in ten equal annnal payments of one hundred and sixty-four dollars and fifty cents. This is done in order to avoid the idea of compound interest, which is inimical to the principles of common law though not of equity, as Mr. J. Ryerson justly states. But no power on earth can prevent this problem, and that of its converse mentioned by Professor McLellan in the January issue of the *Journal*, from coming under the principles of compound interest; inasmuch as the yearly payment is made with two objects in view, namely, to discharge one year's interest of the principal, and cancel a portion of the debt. Upon this view of the case has Mr. J. Ryerson proceeded, and has ascertained the correct rate per cent. But this is at compound interest, which it is the object of this communication to prove.

Without further preface then, I shall repeat the problem given by Professor McLellan and that of its converse given by Mr. Cameron, and shall employ the same principles to resolve them both.

1st.

A man bought a farm for \$5000, and agreed to pay principal and interest (6 per cent.) in four equal annual payments. Find the annual payment.

2nd.

A. lends B. \$1000, payable in ten annual instalments of \$160 each. What rate per cent. does B. pay for his money?

Solution.

Let a be the principal, b the annual payment, and r the rate per unit.

$a(1+r)-b =$ the principal after the 1st payment.

$a(1+r)^2-b(1+r)-b =$ the principal after the second payment.

$a(1+r)^3-b(1+r)^2-b(1+r)-b =$ principal after the third payment.

Generally

$$a(1+r)^n-b(1+r)^{n-1}-b(1+r)^{n-2}-b(1+r)^{n-3} \ldots\ldots\ldots\ldots b=0$$

$$=a(1+r)^n-b\left\{(1+r)^{n-1}+(1+r)^{n-2}+(1+r)^{n-3}+\ldots\ldots\ldots\ 1\right\}=0.$$

Showing that the amount of the principal for the given time, at compound interest, is equal to the sum of the amounts of each payment for periods of one year, two years, three years, . . . less than the given time, plus the payment, proving that these problems belong to compound and not to simple interest. In fact, Professor McLellan positively states that the first belongs to compound interest. The second must also belong to compound interest, as it is only the converse of the first. In the 1st problem b is required and $(1+r)$ given, in the second b is given and $(1+r)$ required.

$$b\left\{(1+r)^{n-1}+(1+r)^{n-2}+(1+r)^{n-3}+\ldots\ldots 1\right\}=b\frac{(1+r)^n-1}{r}$$

$$\text{Hence } a(1+r)^n = b.\frac{(1+r)^n-1}{r}.$$

$$\therefore b=\frac{ar\cdot(1+r)^4}{(1+r)^n-1}=\frac{5000\times\cdot06\times(1+.06)^4}{(1\cdot06)^4-1}=\$1442\cdot944.$$

In the second problem we have, according to the same principle,

$$1000(1+r)^{10}-160\left\{(1+r)^9+(1+r)^8+\ldots\ldots\ldots 1\right\}=0,$$

$$\text{or } (1+r)^{10}-\tfrac{4}{25}\left\{(1+r)^9+(1+r)^8+(1+r)^7+(1+r)^6+(1+r)^5 +(1+r)^4+(1+r)^3+(+r)^2+(1+r)+1\right\}=0.$$

A beautiful geometrical progression of eleven terms, commencing at unity, in which the last term is the four twenty-fifths of the sum of the other ten. By summing these ten, we have $(1+r)^{10}-\frac{4}{25}\left\{\frac{(1+r)^{10}-1}{r}\right\}=0$, from which we find $r=\frac{\frac{4}{25}(1+r)^{10}-1}{(1+r)^{10}}$ and by adding unity to both sides, we have $(1+r)=1+\frac{4}{25}\times\frac{(1+r)^{10}-1}{(1+r)^{10}}$ and by clearing and transposition $(1+r)^{11}-1\cdot16(1+r)^{10}=-\cdot16$, and by employing Newton's rule of trial and error we obtain $1+r=1\cdot09606998$, the same answer which the Mathematical Editor of the *Journal* and Mr. Jesse Ryerson discovered. By adopting the same process we find that the Building Society charges the usurious sum of $10\frac{1}{4}$ per cent. for their money. The general formula for such problems is, $(1+r)^{n+1}-\frac{a+b}{a}.(1+r)^n=-\frac{b}{a}$, assuming, as we have already done, a, the capital, b the yearly payment, r the rate per unit, and n, the No. of years.

I am, Sir, your obt. servt.

D. C. SULLIVAN.