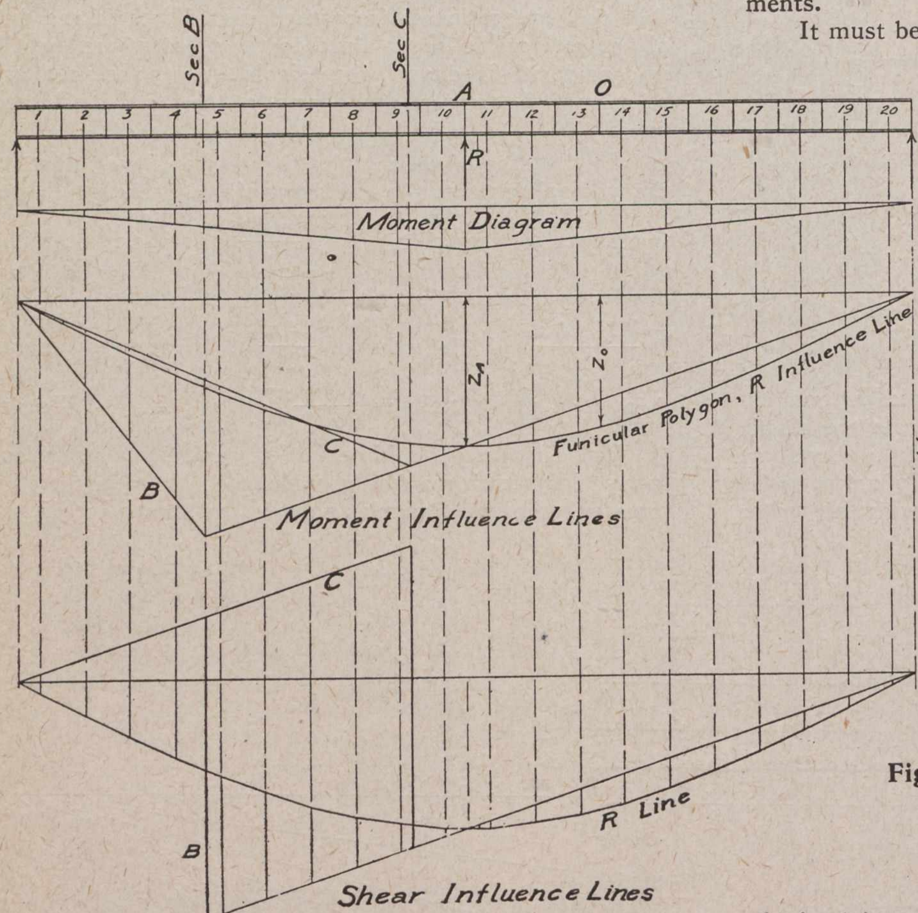
The centres of the different divisions of the beam are projected down through the moment diagram and the corresponding moments are laid off algebraically in order on the load line of the force polygon. A pole is then chosen at any convenient distance, p, from the load line and the rays are drawn. The funicular polygon $A''B'$ is drawn with its sides parallel to the rays of the force polygon just as though the moments were forces acting at the centres of gravity of the division of the beam. A consideration of the geometry of the two figures will show at once that the ordinate z_0 is equal to $\sum_0^B Mx$ divided by p, that is $\sum_0^B Mx = pz_0$. This may be seen from the two shaded triangles Ofg and $o'f'g'$ which are similar



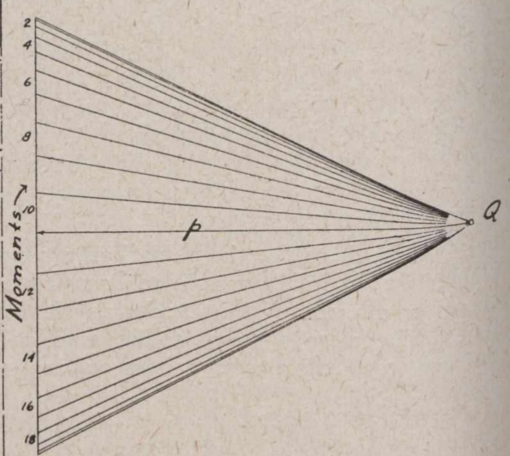


Fig. 3.—Graphical Determination of Reactions, Moments and Shears for Continuous Beams.

because their sides were made parallel. $M_4 : a = p : x_4$ or $a = \frac{M_4 x_4}{p}$. The ordinate z_0 is therefore equal to the summation of the values of $\frac{Mx}{p}$ from O to B. Since this relation is true for any point, a curve drawn tangent to the funicular polygon is the elastic curve itself and the deflection of the beam at any point is given by the expression

$$d_0 = \Delta_0 y = pz_0 \frac{\Delta s}{EI}.$$

The value of p is measured by the scale of the load line of the force polygon and z_0 by the scale of the beam diagram AB. From Fig. 2 the following results were obtained:

$$z_A = A''A' = .506L$$

$$d_A = 7\,PL \times .506L \times \frac{\Delta s}{EI}$$

$$\frac{\Delta s}{EI} = \frac{L}{10\,EI_B}$$

$$d_A = \frac{3.542\,PL^3}{10\,EI_B} = 0.3542\frac{PL^3}{EI_B}$$

Church's "Mechanics of Internal Work" gives the value $.3540\,\frac{PL^3}{EI_B}$ for the deflection of the same beam. Eighteen or twenty divisions of the beam or rib are enough for ordinary cases and the scale need not be made large to bring the error within one or two per cent.

Exactly the same method may be applied to ribs with curved or polygonal axes. The moments for any loading are laid off algebraically on the load line and the funicular polygon is constructed on the lines formed by projecting down the centres of the divisions of the rib. The curve obtained is, of course, only the curve of the y displacements.

It must be apparent that the x-displacements may be obtained in exactly the same manner by projecting the centre points horizontally and constructing a funicular polygon with sides perpendicular to the rays of the force polygon used for the y-displacements. The application to curved arched ribs will be shown in another article.

This simple method of determining displacements may be used to great advantage in analyzing statically indeterminate ribs or beams. It is used as a means of applying the theory of virtual displacements, and Maxwell's Theorem of reciprocal displacements is also employed.

A statically indeterminate frame may be considered as a statically determinate frame with certain redundant forces acting on it. These forces or reactions must be such that when the frame is loaded its deflections at certain points are as desired. That is, the deflections at points of rigid supports must be zero, etc. The deflections of a frame when acted upon by a number of forces are equal to the sums of the deflections produced by the forces acting independently and this makes a simple analysis of a complex structure possible.

Consider the simple case of the cantilever previously treated and let an unyielding support be placed under the