Therefore

$$z\{he(\theta^2-\phi^2 z)\}^2=(k^2-t^2y)^2+(g^2-y)^3+(g^2-y)\{4kty-2g(k^2+t^2y)\}.$$

But $A'=he(\theta^2-\phi^2 z)$. Therefore, by (36),

$$\frac{1}{y}\left\{\frac{g^2-y}{20}(5g^2+15y-p_4)-g(k^2-t^2y)\right\}^2=(k^2-t^2y)^2+(g^2-y)^3$$
$$+(g^2-y)\{4kty-2g(k^2+t^2y)\};$$

or, arranging according to the powers of y,

$$\begin{aligned}&25y^3-y^2\left(16t^4+56gt^2-64kt+\frac{6}{5}p_4+35g^2\right)\\&-y\left\{8gt^2\left(g^2-\frac{1}{5}p_4\right)-32k^2t^2+\left(g^2-\frac{1}{5}p_4\right)\left(5g^2+\frac{1}{5}p_4\right)+8gk^3-16g^4\right\}\\&-g^2\left(g^2-\frac{1}{5}p_4\right)^2+8gk^2\left(g^2-\frac{1}{5}p\right)-16k^4=0.\end{aligned}\tag{38}$$

This is the second equation between the unknown quantities y and t.

§21. We may now either eliminate t from the two equations (37) and (38) so as to obtain an equation

$$F(y)=0$$

whose coefficients are rational functions of the coefficients of the quintic to be solved, or we may eliminate y so as to obtain an equation

$$\psi(t)=0$$

whose coefficients are rational functions of those of the quintic to be solved. In the former case, let the commensurable root y of the equation $F(y)=0$ be found. Then, by (37) and (38), t is known. In the latter case, let the commensurable root t of the equation $\psi(t)=0$ be found. Then, by (37) and (38), y is known. When y and t have thus been found, we find B and $B'\sqrt{z}$, exactly as in §12, from the equations (34), or B can more readily be found from the second of equations (6). Then

$$\begin{aligned}u_1^5&=B+B'\sqrt{z}+\sqrt{\{(B+B'\sqrt{z})^2-(u_1u_4)^5\}}\\&=B+B'\sqrt{z}+\sqrt{\{(B+B'\sqrt{z})^2-(g+a\sqrt{z})^5\}}.\end{aligned}$$

Therefore

$$\begin{aligned}x&=u_1+u_4+u_2+u_3\\&=[B+B'\sqrt{z}+\sqrt{\{(B+B'\sqrt{z})^2-(g+a\sqrt{z})^5\}}]^{\frac{1}{5}}\\&+[B+B'\sqrt{z}-\sqrt{\{(B+B'\sqrt{z})^2-(g+a\sqrt{z})^5\}}]^{\frac{1}{5}}\\&+[B-B'\sqrt{z}+\sqrt{\{(B-B'\sqrt{z})^2-(g-a\sqrt{z})^5\}}]^{\frac{1}{5}}\\&+[B-B'\sqrt{z}-\sqrt{\{(B-B'\sqrt{z})^2-(g-a\sqrt{z})^5\}}]^{\frac{1}{5}}.\end{aligned}$$

It need scarcely be pointed out that since $y=a^2z$, $a\sqrt{z}$ is known.