

of two stations, the temperature being t , the barometer reading at lower station b , and at the upper station $b - \frac{1}{10}$. Hence, by (iii.),

$${}_b N_t = A_t \log \frac{b}{b - \frac{1}{10}}.$$

Also R being the reduction, (iii.) may be written

$$Z = A_t \log \frac{\beta + R}{\beta}.$$

Combining these, we get

$$\log \left(1 + \frac{R}{\beta} \right) = \frac{Z}{{}_b N_t} \log \frac{10b}{10b - 1};$$

hence,
$$1 + \frac{R}{\beta} = \left(\frac{10b}{10b - 1} \right) \frac{Z}{{}_b N_t} = \left(1 - \frac{1}{10b} \right) - \frac{Z}{{}_b N_t}$$

$$= 1 + \frac{Z}{{}_b N_t} \cdot \frac{1}{10b} + \frac{1}{1.2} \cdot \frac{Z}{{}_b N_t} \cdot \frac{Z}{{}_b N_t} + 1 \cdot \frac{1}{10b} \Big| ^2 + \dots$$

by the binomial theorem.

$$\therefore R = \beta \left(\frac{Z}{{}_b N_t} \cdot \frac{1}{10b} + \frac{1}{1.2} \cdot \frac{Z}{{}_b N_t} \cdot \frac{Z}{{}_b N_t} + 1 \cdot \frac{1}{10b} \Big| ^2 + \dots \right) \quad (\text{iv.})$$

Formula (i.) is deduced from (iv.), by neglecting all terms beyond the first; and making $b = 30$ inches, if used with Table XVI.; but, if used with Table XIX., b may be any reading within the range of the table, and ${}_b N_t$ the corresponding number from the table.

Although (i.) is sufficiently accurate for small heights, it is evident, on comparing it with the full formula (iv.), that it becomes more and more inaccurate as the height increases.

If, in (i.), the reduced height B , were substituted for the observed height β , the error would be relatively less; for Laplace's formula may also be expanded in the form

$$R = B \left(\frac{Z}{{}_b N_t} \cdot \frac{1}{10b} - \frac{1}{1.2} \cdot \frac{Z}{{}_b N_t} \cdot \frac{Z}{{}_b N_t} - 1 \cdot \frac{1}{10b} \Big| ^2 + \dots \right) \quad (\text{v.})$$