$\therefore\ x+\frac{2\pi x}{180}=23\frac{3}{7}$. Whence $x=22\frac{1}{2}$.

2. (*a*) Book Work.

(*b*) Given $2\sin^2 45^\circ = 1$ and $\sin 30^\circ = \frac{1}{2}$; find $\tan 15^\circ$.

By the formula $\cos\theta = \sqrt{1-\sin^2\theta}$ we obtain $\cos 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$. Hence $\tan 45^\circ = 1$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$$\text{Tan } 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = 2-\sqrt{3}$$

3. (*a*) Book work.

(*b*) If $\cos(A-C)\cos B = \cos(A-B+C)$ show that tan A, tan B, and tan C are in harmonical progression.

$(\cos A\cos C + \sin A\sin C)\cos B = \cos(A+C)\cos B + \sin(A+C)\sin B$; *i.e.*, $\cos A\cos C\cos B + \sin A\sin C\cos B = \cos A\cos C\cos B - \sin A\sin C\cos B + \sin A\cos C\sin B + \cos A\sin C\sin B$. Cancel, collect like terms and divide throughout by $\sin A\sin B\sin C$;

and $\frac{2}{\tan B} = \frac{1}{\tan A} + \frac{1}{\tan C}$. $\therefore\ \frac{1}{\tan A}, \frac{1}{\tan B}$ and $\frac{1}{\tan C}$ are in A. P.

Hence tan A, tan B and tan C are in H.P.

4. Prove the following identities:

(*a*) $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$$\tan\frac{x}{2} = \frac{\sin^x/_2}{\cos^x/_2} = \frac{2\sin^{2x}/_2}{2\sin^x/_2\cos^x/_2} = \frac{1-\cos x}{\sin x}$$

(*b*) $\cos A + \cos 3A + \cos 5A + \cos 7A = 4\cos A\cos 2A\cos 4A$.

Combine the 1st and 4th; and the 2nd and 3rd terms of the left hand member, and it becomes $2\cos 4A\cos 3A + 2\cos 4A\cos A = 2\cos 4A(\cos 3A + \cos A) = 4\cos A\cos 2A\cos 4A$.

(*c*) $\cos(A+B) - \sin(A-B) = 2\sin(^\pi/_4 - A)\cos(^\pi/_4 - B)$.

Since the cosine of an angle is equal to the sine of its complement, the left-hand member of the equation may be written $\sin(\pi/2 - A - B) - \sin(A - B)$. This expression is equal to twice the product of the cosine of the half sum and the sine of the half difference.

$\therefore\ = 2\cos(\pi/4 - B)\sin(\pi/4 - A)$.

5. Define the logarithm of a number and prove

(*a*) $\log_a\sqrt{^n/_m} = \frac{1}{2}(\log_a n - \log_a m)$.

If $a^x = y$ then the definition of a logarithm is that x is the logarithm of y the base a; and this relation is otherwise indicated by writing $x=\log_a y$.

Let $a^x = m$, and $a^y = n$.

Then from the definition of a logarithm $x=\log_a m$, and $y=\log_a n$.

And $\sqrt{\frac{n}{m}} = \sqrt{\frac{a^y}{a^x}} = a^{\frac{1}{2}(y-x)}$

$\therefore\ \log_a\sqrt{\frac{n}{m}} = \frac{1}{2}(y-x) = \frac{1}{2}(\log_a n - \log_a m)$.

(*b*) Prove $6\log_a\frac{2}{3} + 4\log_a\frac{9}{10} + 2\log_a\frac{25}{6} = 0$