

is worth $\frac{1}{3}$ of \$114 or \$152 per carat, and the diamond is $\therefore$ worth \$608.

4. Solve the equations

$$20x^{n+1} - 21x^n + 1 = 0 \quad (1)$$

$$26x^{2n+1} - 27x^{2n} + 1 = 0 \quad (2)$$

Multiply (1) by $13x^n$, (2) by 10 and subtract;

$$\text{then } 3x^{2n} - 13x^n + 10 = 0$$

$$\therefore x^n = 1 \text{ or } 3\frac{1}{3}$$

and substituting for x^n in (1) we have

$$20x \times 3\frac{1}{3} - 21 \times 3\frac{1}{3} + 1 = 0$$

$$200x = 207$$

$$\therefore x = 1.035$$

5. A railway train after travelling an hour meets with an accident, after which it proceeds at three-fourths of its former rate, and arrives 9 minutes late. If the detention had taken place 5 miles further on the train would have arrived 3 minutes sooner than it did. Find the original rate of the train and the distance travelled.

A B C D

A, starting point; B, when detention occurred; C, 5 miles further on; and D, its destination.

Now, since the time required to go any distance is inversely proportioned to the rate, and since the rate from B to D is $\frac{3}{4}$ the ordinary rate, $\therefore$ the time from B to D is $\frac{4}{3}$ the ordinary time; that is, the time has been increased by one-third on account of the accident, and the time has been increased by 9 minutes; $\therefore$ the ordinary time of going from B to D is 27 minutes. Similarly, if the accident had occurred at C, we should get the ordinary time from C to D to be 18 minutes; hence the ordinary time from B to C (5 miles) is 9 minutes; that is, the rate is $33\frac{1}{3}$ miles per hour. Also AB being one hour's journey is $33\frac{1}{3}$ miles, and BD requiring 27 minutes must be 15 miles; $\therefore$ whole distance is $48\frac{1}{3}$ miles.

HINTS FOR SOLVING DEDUCTIONS.

In solving exercises upon Euclid, there are two golden rules to be remembered—(1) if the exercise be a *problem*, strictly so called, that is to say, if some line or figure be required to be drawn, *imagine it done, and find*

all the conditions that result from so doing; (2) if the exercise be a *theorem*, *examine all the conditions given, and those which will come into play if the theorem be true, and then endeavor to combine and connect them*. Thus, suppose we had to show how to describe an equilateral triangle on a given line. This being a *problem*, *suppose* on the line AB an equilateral triangle ABC to be described, then by definition AB equal AC, therefore a circle with A as centre and radius AB will pass through C, similarly a circle whose centre is B and radius AB will pass through C; hence we learn that C is the intersection of the two equal circles, and so we can start the construction as in Euclid I. 1.

Again, as an example of a *theorem*, we may prove that the exterior angle of a triangle is equal to the two interior and opposite angles. Supposing this theorem to be true, since the exterior angle is equal to the sum of the two interior, if we draw a line cutting off from it an angle equal to one of the interior angles, the remainder must equal the other interior angle. Now we know that if this line be drawn parallel to the opposite side, the alternate angles will be equal, and it is clear that the remaining angles are also equal, being interior and exterior angles on two parallel lines, therefore the theorem is true. Hence the construction as in Euclid I. 32.

Suppose we had this *theorem* set, prove that if two tangents be drawn from an exterior point to a circle they are equal. *Suppose* that they are equal join the external point with the centre, and the centre with the points of contact, we then have two triangles, the three sides of one being equal to the three sides of the other each to each, therefore the angles of the one are equal to the angles of the other, therefore the right angles are equal. But this we knew before; we start from this and retracing our steps can show that since the radii are equal and the line from