II.—AS RELATING TO ANNUITIES.

Let t = the *true interest* on 1 for single period of the annuity.
r = $(1 + t)$ the *amount* of 1 for one period.
n = the *number* of periods or instalments of the annuity.
A = the *amount* of the whole of the annuities at the end of n periods.
V = the *principal*; or the *present value* of the annuity of \$$a$ or £a for n periods (each annuity being assumed to be payable at the *end* of its own period)
a = the periodic *payment* or *annuity*.
S = the *sinking fund*, or the excess of the annuity (a) over the interest on the principal for one period (Vt). $S = a - Vt$.
D = the *present value* of a *deferred annuity*, first payment being made at the end of $d + 1$ periods.
d = the *number* of periods during which the annuity is *deferred*. Then

To find the *present value* of any annuity.

(5) $V = \frac{a}{t}\left(1 - \frac{1}{r^n}\right)$ or $\log. V = \log.\left(1 - \frac{1}{r^n}\right) + \log. a - \log. t$; $\log. \frac{1}{r^n} = 0 - n \times \log. r$.

or (6) $V = \frac{a(r^n - 1)}{tr^n}$ or $\log. V = \log. (r^n - 1) + \log. a - \log. t - n \times \log. r$.

or (7) $V = \frac{a}{t} - \frac{a}{tr^n}$ or $\log. \frac{a}{t} = \log. a - \log. t$; and

$\log. \frac{a}{tr^n} = \log. a - \log. t - n \times \log. r$.

To find the *amount* of any annuity.

(8) $A = \frac{ar^n}{t}\left(1 - \frac{1}{r^n}\right)$ or $\log. A = \log.\left(1 - \frac{1}{r^n}\right) + \log. a + n \times \log. r - \log t$.

or (9) $A = \frac{a}{t}(r^n - 1)$ or $\log. A = \log. (r^n - 1) + \log. a - \log. t$.

or (10) $A = \frac{ar^n}{t} - \frac{a}{t}$ or $\log. \frac{ar^n}{t} = \log. a + n \times \log. r - \log. t$; and

$\log. \frac{a}{t} = \log. a - \log. t$.

To find the *annuity* which a given sum V will purchase.

(11) $a = \frac{Vt}{1 - \frac{1}{r^n}}$ or $\log. a = \log. V + \log. t - \log. \left(1 - \frac{1}{r^n}\right)$

or (12) $a = \frac{Vtr^n}{r^n - 1}$ or $\log. a = \log. V + \log. t + n \times \log. r - \log. (r^n - 1)$.

The period and rate being given, to find what *annuity* it would take *to amount* to a given sum (A) at the end of a certain number of periods.

(13) $a = \frac{At}{r^n\left(1 - \frac{1}{r^n}\right)}$ or $\log. a = \log. A + \log. t - n \times \log. r - \log. \left(1 - \frac{1}{r^n}\right)$

or (14) $a = \frac{At}{r^n - 1}$ or $\log. a = \log. A + \log. t - \log. (r^n - 1)$