

Fig. 2 shows two rigid supports and a wire suspended between these supports. Let L_0 represent the length of the wire under maximum loading; i.e., under a loading equal to the resultant of the dead load of the wire plus the weight of $\frac{1}{2}$ inch of ice and a wind pressure of 8 lbs. per square foot of projected area of ice-covered wire at a temperature of 0° F.

Evidently this wire is undergoing a change in length at some other temperature and different loading. A simultaneous change of temperature and loading is more complicated, but by letting the loading remain constant for the time being, calculating the change in length due to the temperature only and finally combining this with the change due to the different loading, the total change in the length is easily found.

Let L_1 (Fig. 2) be this changed length of the wire. The effect of the temperature is always partly counter-balanced by the effect of the change in tension. A higher temperature will stretch the wire and increase the sag. This increased sag in turn will diminish the tension and cause a shortening in the length. The algebraic sum of these changes will be the actual change in the length. The author has seen several charts which do not take account of this change due to the tension, caused by the change of temperature. This factor is quite an appreciable one, even with moderate sags.

Let $L_0 \alpha t$ be the linear expansion due to the rise in temperature t , then $L_0 (1 + \alpha t)$ would be the total length of the wire, providing the tension would remain the same. Since, however, the tension becomes less with the increase in length, the shortening has to be added algebraically to the above value $L_0 (1 + \alpha t)$, and the actual length is thus found. In order to derive an expression for this shortening in the length, we have to employ following fundamental formula:

$$\text{Modulus of elasticity } E = \frac{\text{unit stress}}{\text{unit strain}} = \frac{P_0/a}{e_0/L_0} = \frac{P_1/a}{e_1/L_1}$$

and by transposing $e_0 = \frac{P_0 L_0}{a E}$ and $e_1 = \frac{P_1 L_1}{a E}$ where

e_0 and e_1 represent the linear deformation due to the tension P_0 and P_1 respectively.

The total difference is

$$L_1 - L_0 = L_0 \alpha t + e_1 - e_0; \text{ or, } L_1 - L_0 = L_0 \alpha t + \frac{P_1 L_1 - P_0 L_0}{a E} \quad (2)$$

Transposing: $P_1 L_1 = L_1 a E - L_0 a E - L_0 \alpha t a E + P_0 L_0$

Dividing by L_1 ; $P_1 = a E - \frac{L_0}{L_1} a E - \frac{L_0}{L_1} \alpha t a E + \frac{L_0}{L_1} P_0$;

$$\text{or, } P_1 = \frac{L_0}{L_1} [P_0 - a E (1 + \alpha t)] + a E$$

The general expression for the length of the wire in

terms of the sag is $L = l + \frac{8 S^2}{3 l} = \frac{3 l^2 + 8 S^2}{3 l}$, and substituting for L_0 and L_1 we have

$$P_1 = \frac{3 l^2 + 8 S_0^2}{3 l^2 + 8 S_1^2} [P_0 - a E (1 + \alpha t)] + a E \quad (3)$$

which is the formula used for compiling Table II. Here we notice that three variables are involved in this equation, P_1 , S_1 and t . By giving t a certain value and assume different values for S_1 , we obtain corresponding values for P_1 which can be plotted. The P_1 values

are plotted on the vertical axis and the S_1 values on the horizontal axis. By repeating this operation and assigning another certain value for t , we obtain a family of curves. This operation renders t a variable parameter.

Since we have two variables now and only one equation, we have to establish some other relation between P_1 and S_1 . This is accomplished by using the well-known formula

$$P_1 = \frac{w l^2}{8 S_1} \quad (4)$$

The value w represents the variable parameter.

Table I. has been worked out for a copper wire of 250,000 C.M. and a span of 600 feet. The values are taken from the reports of the National Electric Light Association, 36th convention:

Table I.—Stranded Hard-drawn Copper Wire of 250,000 C.M.

Area in circular mills	250,000 or $a = .19635$ sq. ins.
Elastic limit	35,000 lbs. per square inch.
Elastic limit	6,870 lbs. per wire.
Allowable tension...	30,000 lbs. per square inch.
Allowable tension... $P_0 =$	5,900 lbs. per wire.
Ultimate strength ..	60,000 lbs. per square inch.
Ultimate strength ..	11,790 lbs. per wire.
Modulus of elasticity	$E = 16 \times 10^6$ lbs. per sq. in.
Coefficient of linear expansion	$\alpha = 96 \times 10^{-7}$
Weight per ft. of wire plus $\frac{1}{2}$ in. ice and 8 lbs. wind.....	$w_0 = 1.788$ lbs.
Weight per ft. of wire, no ice, no wind....	$w_1 = .762$ lbs.
Weight per ft. of wire, no ice, 15 lbs. wind	$w_2 = 1.061$ lbs.
Weight per ft. of wire, $\frac{1}{2}$ in. ice, no wind	$w_3 = 1.440$ lbs.

Above values are substituted in equation (3).

$$P_1 = \frac{3 l^2 + 8 S_0^2}{3 l^2 + 8 S_1^2} [P_0 - a E (1 + \alpha t)] + a E$$

$$l = 600; \quad 3 l^2 = 1,080,000$$

$$S_0 = \frac{w_0 l^2}{8 P_0} = \frac{1.788 \cdot 360000}{8 \cdot 5900} = 13.637; \quad 8 S_0^2 = 1,487.75$$

$$3 l^2 + 8 S_0^2 = 1,080,000 + 1,487.75 = 1,081,488$$

$$a E = .19635 \times 16,000,000 = 3,141,600$$

Table II. gives values for P_1 and S_1 which can be plotted now.

In order to plot equation 4, which furnishes points of intersections with above curves, Table III. has been compiled.

Having plotted all these values for P_1 and S_1 , we obtain an exact picture of the behavior of the wire at various temperatures and under different loadings. It will be noticed that equation (3) is rather lengthy and cumbersome. By making a slight change in this equation the whole computation is materially shortened without affecting the results appreciably. This is accomplished by substituting L_0 for L_1 in equation (2) on the right side

$$\text{only, as shown } L_1 - L_0 = L_0 \alpha t + \frac{P_1 - P_0}{a E} L_0$$

$$\text{Solving for } P_1 \text{ we get } P_1 = P_0 + a E \left[\frac{L_1}{L_0} - 1 - \alpha t \right]$$