

For a semi-circular arch this is practically the same as Rankine's, with the difference of the constant added. It does not increase the thickness so rapidly as his, but gives also an infinite thickness to the straight arch.

There is still need of agreement respecting the rational basis for such formulae, as they differ in principle. They could only be satisfactorily compared by dividing the examples on which they are based into classes, in accordance with their purpose and the conditions of their construction.

To determine the stability of the abutments it is only necessary to continue to the foundations the curves of pressure corresponding to the various cases considered. As we have already had occasion to notice, the abutments require greater strength to resist yielding than to stand after a slight motion has taken place; because the position of the point *K* as determined for unyielding abutments, corresponds to a greater thrust and greater pressures throughout the structure than it does to *A* through the opening of joints in the arch ring. With symmetrical loads the greatest resistance will be required when the arch is completely loaded, and although the effect of partial loading may possibly be greater, this is almost invariably left to the margin of safety. If the curve of pressure strikes outside of the middle third of the base, there is a portion of the abutment whose weight should not be taken into account. (Fig. 7.) The effective width of each course is then only three times the distance from the curve to the back of the abutment; and the curve of pressure should be drawn again, omitting the weight of the remainder. This precaution is especially necessary in the case of segmental arches. When all the cases of loading are considered, the abutment will be undoubtedly stable when the resulting curve is sufficiently within the base to prevent excessive pressure, provided the foundations are thoroughly sound. It will be quite unnecessary to double the thrust from the arch as recommended by some authors, unless indeed this is done to allow for positions of the load which are not considered.

When arches are built in a series, somewhat different conditions arise. When the weight only of the structure is to be taken into account, the pressure on the pier is vertical and the resistance to crushing is the only consideration. For arches carrying a moving load, a pier is the most unfavorably situated when it is between a loaded and an unloaded arch. The reaction at the joints of rupture in the two arches respectively when compounded with the weight of the pier, give the final resultant, which in a pier should always meet the base within the middle third. Before failure can take place however, the thrust of the loaded arch will develop a much greater reaction at the joint of rupture of the unloaded arch than the amount due to its weight only. If the loading increases to the limiting amount, the loaded arch will fail in the ordinary way, but the joints in the unloaded arch will open in the reverse direction, corresponding to a curve of pressure passing through *B* and *C*, and therefore also to a largely increased thrust. Such a thrust should not be counted upon except in the case of small arches used as counterforts, or transverse arches in the interior of an abutment; but it will at least be allowable to consider the pressure at the joint of rupture of the unloaded arch to become equally distributed under the influence of the thrust from the adjoining arch, and so to place *J* at the center of *C'D* in the unloaded arch.

The various theories have now been compared in the endeavor to point out the conditions under which they are applicable, and to ascertain the position of the determining points in the curve of pressure which best accords with them in the various cases considered. It will be unnecessary to proceed further into detail, as the method of drawing the curve, when these points are known, is fully given in works on Graphical Statics. In applying the graphical method, a distinction has in some cases to be made between the "curve of pressure" and the "line of resistance," according to the division of the arch ring into vertical laminae or actual voussoirs. The method of passing from one to the other is given by Prof. Clarke. (19) The distinction between these curves was originally pointed out by Moseley, and is well illustrated by Dubois. (20)

We have not considered the case of unsymmetrical loading, as so few authors touch upon it at all, and those who do differ so widely in opinion. When an arch is loaded symmetrically, it has been maintained that the thrust at the key will remain the same if the load is removed from one half, leaving the load on the other side only. This seems plausible at first sight, as this thrust is undoubtedly sufficient to support the loaded side of the arch; but on continuing the curve on the unloaded side it will often pass entirely out at the extrados in existing structures which could not possibly stand if this were the case. On the other hand, a mean between the values of the thrust for the loaded and unloaded arch is not sufficient to support the loaded side. From a general theorem given by Collignon (21), it would appear that with unsymmetrical loading friction is developed at the key, or in other words the direction of the thrust is inclined towards the less loaded side. This accords also with the direction of the tangent at the centre of an unequally loaded catenary. In the case of an arch with an engine load on one side only, an inclination of 5° to 10° in the line of