7. (1). Let m be the common root, then

$$am^2+bm+c=0 \quad (1).$$
$$pm^2+qm+r=0 \quad (2).$$

Multiply (1) by r, (2) by c and subtract

$$\therefore (ar-cp)m^2 = (cq-br)m \quad (3).$$

again, multiply (1) by p, (2) by a and subtract

$$\therefore (pb-aq)m = ar-pc$$
$$ar-pc = (pb-aq)m \quad (4).$$

Multiply (3) by (4)

$$\therefore (ar-cp)^2m^2 = (cq-br)(pb-aq)m^2$$

$\therefore$ the condition required is

$$(ar-cp)^2 = (cq-br)(pb-aq)$$

(2). Let m, n be the roots of (1) then $-m$, $-n$ are the roots of (2)

$$\therefore m+n = -\frac{b}{a},\ mn = \frac{c}{a}$$
$$-m-n = -\frac{q}{p},\ mn = \frac{r}{p}$$
$$\therefore \frac{b}{a} = \frac{q}{p}. \quad \therefore \frac{a}{p} = \frac{b}{q}$$

also $\dfrac{c}{a} = \dfrac{r}{p} \quad \therefore \dfrac{a}{p} = \dfrac{c}{r}$

$$\therefore \frac{a}{p} = \frac{b}{q} = \frac{c}{r}$$

8. (1). $x = 2\frac{8}{9}$.

(2). $-1 \pm 2\sqrt{3}$

(3). When cleared of fractions the equation reduces to

$$(1-2x)\sqrt{1+2x} + (1+2x)\sqrt{1-2x} = 2x$$
$$\therefore \sqrt{1-2x}\sqrt{1+2x}\,(\sqrt{1-2x}+\sqrt{1+2x}) = 2x$$

square both sides.

$$\therefore (1-4x^2)(2+2\sqrt{1-4x^2}) = 4x^2 \quad (1)$$
$$2(1-4x^2) + 2\sqrt{1-4x^2}\,(1-4x^2) = -(1-4x^2)+1$$
$$\therefore 2\sqrt{1-4x^2}\,(1-4x^2) + 3(1-4x^2) = 1$$

substitute y for $\sqrt{1-4x^2}$, and we have

$$2y^3+3y^2-1=0$$
$$\therefore (y+1)(y+1)(2y-1) = 0$$
$$\therefore y = -1, \text{ or } \tfrac{1}{2}$$
$$\therefore \sqrt{1-4x^2} = -1,\ \therefore x=0$$
$$\text{also } \sqrt{1-4x^2} = \tfrac{1}{2}$$
$$\therefore x = \tfrac{1}{4}\sqrt{3}$$

or thus; (1) reduces to

$$(1-4x^2)^{\frac{3}{2}} = (6x^2-1$$
$$\therefore (1-4x^2)^3 = (6x^2-1)^2$$

expanding these we get

$$x = 0, \text{ or } \tfrac{1}{4}\sqrt{3}$$

9. $$\frac{x^2(n-1)^2-2x(n-1)^2+(n+1)^2}{x^2(n-1)^2+2x(n-1)^2+(n+1)^2} = p$$

$$\therefore x^2(n-1)^2(1-p) - 2x(n-1)^2(1+p) + (n+1)^2(1-p) = 0$$

$$\therefore x = \frac{1+p}{1-p}$$

$$\pm \frac{\sqrt{(n-1)^2(1+p)^2-(n+1)^2(1-p)^2}}{(n-1)(1-p)}$$

The num. of this last fraction becomes

$$2\sqrt{n(n-p)\left(p-\frac{1}{n}\right)}$$

and in order that x may be real,

$n(n-p)\left(p-\dfrac{1}{n}\right)$ must be positive.

$\therefore$ since n is positive, $n-p$ and $p-\dfrac{1}{n}$ must be both positive or both negative.

$\therefore p$ is either $< n$ and $> \dfrac{1}{n}$ or $> n$ and $< \dfrac{1}{n}$ and $\therefore$ always lies between n and $\dfrac{1}{n}$

10. The H. M. between $H-a$, $H-b$ is

$$\frac{2(H-a)(H-b)}{H-a+H-b} = \frac{2H^2-2(a+b)H+2ab}{2H-\overline{a+b}} \quad (1)$$

and since $H = \dfrac{2ab}{a+b}$ $\therefore H(a+b) = 2ab$

$$\therefore (1) = \frac{2H^2-(a+b)H}{2H-(a+b)} = H.$$

11. (1). $d = -\frac{1}{6}$.

$\therefore$ 37th term $= a + 36d = 0$.

sum of 31 terms $= 108\frac{1}{2} =$ sum of 42 terms.

(2). Common ratio $= \frac{3}{5}$

$$\therefore \text{Sum} = 3\tfrac{1}{3}\,\frac{1-(\frac{3}{5})^n}{1-\frac{3}{5}}$$

(3). Common ratio is $-\frac{4}{10}$

$\therefore$ sum to infinity $= \frac{5}{7}$