to him; so that he next receives $\frac{4}{104}$ of 100 less $\frac{4}{104}$ of the unknown percentage already received. Hence

$$\text{percentage} + \tfrac{4}{104} \text{ of } 100 - \tfrac{4}{104} \text{ of percentage} = \tfrac{375}{100},$$
$$\tfrac{100}{104} \text{ of percentage} = \tfrac{375}{100} - \tfrac{400}{104},$$
$$\text{percentage} = \tfrac{104}{100} \text{ of } \tfrac{15600}{100 \times 104}$$
$$= 2\tfrac{1}{4}.$$

10. (1) Area $= \frac{112\frac{1}{2}}{360}$ of $(80)^2 \times 3.14159 = 6283{\cdot}18$ sq. yds.

Arc $= \frac{112\frac{1}{2}}{360}$ of $160 \times 3{\cdot}14159 = 157{\cdot}0795$ yds.

For ratio $\frac{112\frac{1}{2}}{360}$ see Euc. Prop. 33, Bk. VI.

(2) Let $ABCD$ be the quadrilateral,—$AB = 3$, $BC = 5$, $CD = 6$, $DA = 4$. Draw AE parallel to BC, meeting CD in E. Then the sides of the triangle ADE being 3, 4, 5, since $5^2 = 3^2 + 4^2$, ADE is a right angle; $\therefore$ area $= \frac{1}{2}(3+6) \times 4 = 18$.

ALGEBRA.

TIME—THREE HOURS.

Examiner—J. A. McLellan, LL.D.

Note.—*Ten questions reckoned a full paper.*

1. Prove that $2\{(a-b)^7 + (b-c)^7 + (c-a)^7\} = 7(a-b)(b-c)(c-a)\{(a-b)^4 + (b-c)^4 + (c-a)^4\}$.

2. Extract the square root of $ab - 2a\sqrt{(ab - a^2)}$, and find the simplest real forms of the expression
$\sqrt{(3 + 4\sqrt{-1})} + \sqrt{(3 - 4\sqrt{-1})}$.

3. Solve the equations:

(1). $2x^4 + x^3 - 11x^2 + x + 2 = 0$.

(2). $x^2 + y^2 + z^2 = a^2$
$yz + zx + xy = b^2$
$x + y - z = c$.

(3). $\sqrt{(x^2 + 5x + 4)} + \sqrt{(x^2 + 3x - 4)} = x + 4$.

4. Prove that the number of positive integral solutions of the equation $ax + by = c$ cannot exceed $\frac{c}{ab} + 1$.

In how many ways may £11 15s. be paid in half-guineas and half-crowns.

5. If $xy = ab(a + b)$, and $x^2 - xy + y^2 = a^3 + b^3$, shew that $\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{b} - \frac{y}{a}\right) = 0$.

6. Given the sum of an arithmetical series, the first term, and the common difference, shew how to find the number of terms. Explain the negative result. *Ex.* How many terms of the series 6, 10, 14, &c., amount to 96?

7. Find the relation between p and q, and $x^3 + px + q = 0$ has two equal roots, and determine the values of m which will make $x^2 + max + a^2$ a factor of $x^4 - ax^3 + a^2x^2 - a^3x + a^4$.

8. In the scale of relation in which the radix is r shew that the sum of the digits divided by $r - 1$ gives the same remainder as the number itself divided by $r - 1$.

9. Assuming the Binomial Theorem for a positive integral index, prove it in the case of the index being a positive fraction.

Shew that the sum of the squares of the co-efficients in the expansion of $(1 + x)^n$ is $\underline{|2n} \div (\underline{|n})^2$, n being a positive integer.

10. Sum the following series:—

(1.) $1 + 3x + 5x^2 + 7x^3 +$ &c. to n terms.

(2). $\frac{1}{3 \times 8} + \frac{1}{8 \times 13} +$ &c. to n terms, and to infinity.

11. Shew that $\begin{vmatrix} bc & -ac, & -ab \\ b^2-c^2 & a^2+2ac, & -a^2-2ab \\ c^2, & c^2, & (a+b)^2 \end{vmatrix}$ is divisible by $abc(a + b + c)$.

SOLUTIONS.

1. Put x, y, z for $a - b, b - c, c - a$, and then shew that $x + y + z$ is a factor of $2(x^7 + y^7 + z^7) - 7xyz(x^4 + y^4 + z^4)$. But $x + y + z = a - b + b - c + c - a = 0$. Hence given expression is an identity.

2. (1) $= a - \sqrt{ab - a^2}$.

(2) $= 2 + \sqrt{-1} + 2 - \sqrt{-1} = 4$.

3. (1) $= (2x^2 - 5x + 2)(x^2 + 3x + 1) = 0$; or $(x - 2)(2x - 1)(x^2 + 3x + 1) = 0$; *i.e.*, $x = 2$, or $\frac{1}{2}$, or $\frac{-3 \pm \sqrt{5}}{2}$. Or it may be solved as a reciprocal equation.

(2) From first two equations, $x + y + z = \pm\sqrt{a^2 + 2b^2}$; also, $x + y - z = c$. $\therefore z = \frac{1}{2}\{\pm\sqrt{a^2 + 2b^2} - c\}$, and thence x and y may be found.

(3) $\sqrt{x + 4}$ is a factor, giving $x = -4$ as one root. Dividing through by $\sqrt{x + 4}$, we have, to find other roots, $\sqrt{x + 1} + \sqrt{x - 1} = \sqrt{x + 4}$, or $2x + 2\sqrt{x^2 - 1} = x + 4$, or $x = \frac{-4 \pm 2\sqrt{19}}{3}$.

4. (1) Book-work.

(2) Let x be number of half guineas; y, number of half crowns. Then $10\frac{1}{2}x + 2\frac{1}{2}y = 235$, $21x + 5y = 470$. Also, $21 \times 1 - 5 \times 4 = 1$; $\therefore 21 \times 470 - 5 \times 1880 = 470$; $\therefore 21(x - 470) + 5(y + 1880) = 0$, or with usual notation, $-5t = x - 470$, $21t = y + 1880$; thence $x = 20, 15, 10, 5$, or 0; and corresponding values of y are 10, 31, 52, 73, 94.

5. From given equations, $\frac{x}{y} - 1 + \frac{y}{x} = \frac{a}{b} - 1 + \frac{b}{a}$, or $\left(\frac{x}{y}\right)^2 - \left(\frac{a}{b} + \frac{b}{a}\right)\frac{x}{y} + 1 = 0$, or $\left(\frac{x}{y} - \frac{a}{b}\right)\left(\frac{x}{y} - \frac{b}{a}\right) = 0$, or $\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{b} - \frac{y}{a}\right) = 0$.

6. The values of n are 6 and -8. The first has reference to the series of 6, 10, ... 26. The negative value has reference to the series obtained by starting with 26 and counting backwards 8 terms, *i. e.*, the series -2, 2, 6 ...... 26.

7. (1). Let a, a, c be the roots, then from relations between roots and co-efficients (See May number of the Journal), $2a + c = 0$, $a^2 + 2ac = p$, $-a^2c = q$; $\therefore c = -2a$; $\therefore -3a^2 = p$, $2a^3 = q$, or $\left(\frac{q}{2}\right)^2 = \left(-\frac{p}{3}\right)^3$.

(2). Let $x^2 + pax + a^2$ be the other factor, then multiplying $x^2 + pax + a^2$ by $x^2 + max + a^2$ and equating the co-efficients with those of corresponding powers of $x^4 - ax^3 +$ &c., we have $m + p = -1$, $mp = -1$, thence $m = \frac{-1 \pm \sqrt{5}}{2}$.

8. Book-work.

9. Let $(1 + x)^n = p_0 + p_1x + \ldots\ldots + p_nx^n$. Then also $(x + 1)^n = p_0x^n + p_1x^{n-1} + \ldots\ldots + p^n$.

And these are identities. Hence if we multiply them together, co-efficients of corresponding powers of x on both sides will be equal. On right hand side co-efficient of x^n is sum of squares of co-efficients. On left hand side, co-efficient of x^n in $(1 + x)^{2n}$ is
$$\frac{2n(2n-1)\ldots\ldots(2n-n+1)}{\underline{|n}} = \frac{\underline{|2n}}{(\underline{|n})^2}.$$

10. (1) Let $S = 1 + 3x + 5x^2 + 7x^3 \ldots + (2n-3)x^{n-2} + (2n-1)x^{n-1}$.

$\therefore Sx = x + 3x^2 + 5x^3 + \ldots + (2n-5)x^{n-2} + (2n-3)x^{n-1} + (2n-1)x^n$.

$\therefore S(1-x) = 1 + 2\{x + x^2 + x^3 + \ldots + x^{n-2} + x^{n-1}\} - (2n-1)x^n$.

$$= 1 + 2\frac{x^n - x}{x - 1} - (2n-1)x^n.$$
$$S = \frac{2x^n - x - 1}{(x-1)^2} - (2n-1)\frac{x^n}{x-1}.$$

(2) Let $S = \frac{1}{3} + \frac{1}{8} + \frac{1}{13} + \ldots\ldots + \frac{1}{5n-2} + \frac{1}{5n+3}$.