THE RELATION WHICH CAN BE PROVED TO SUBSIST BETWEEN THE AREA OF A PLANE TRIANGLE AND THE SUM OF THE ANGLES, ON THE HYPOTHESIS THAT EUCLID'S 12TH AXIOM IS FALSE.

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I propose to prove in the present paper, that, if Euclid's 12th Axiom be supposed to fail in any case, a relation subsists between the area of a plane triangle and the sum of the angles. Call the area A; and the sum of the angles S; a right angle being taken as the unit of measure. Then

$$A = k\,(2-S);$$

k being a constant finite quantity, that is, a finite quantity which remains the same for all triangles. This formula may be considered as holding good even when Euclid's 12th Axiom is assumed to be true; only k is in that case infinite.

Before proceeding with the proof of the law referred to, I would observe, that, while on the one hand Euclid's 12th Axiom is assuredly *not an Axiom* in the proper sense of the term, that is, not a self-evident truth, on the other hand *it has never been demonstrated* to be true. I even feel satisfied, from metaphysical considerations, that a demonstration of its truth is impossible. Legendre's supposed demonstration, which Mathematicians appear to have accepted as valid, was shown by me, in the *Canadian Journal* for November, 1856, to be erroneous.* For the sake of those who may not have the former

* In an Essay on Mathematical Reasoning, appended to his Mathematical Euclid, Dr. Whewell refers to the attempts which have been made to dispense with Euclid's 12th Axiom. "No one," he writes, "has yet been able to construct a system of Mathematical truth by means of Definitions alone, to the exclusion of Axioms; though attempts having this tendency have been made constantly and earnestly. It is, for instance, well known to most readers, that many mathematicians have endeavoured to get rid of Euclid's Axioms respecting straight lines and parallel lines; but that *none of these essays have been generally considered satisfactory.*" The last clause in this statement calls for remark. Sir John Leslie objected to Legendre's reasoning; but on grounds which (as Professor Playfair showed in the *Edinburgh Review*) are altogether frivolous. Playfair maintained that Legendre's proof was satisfactory; and since then, till the publication in the *Canadian Journal* of the article above referred to, mathematicians have—by their silence at least—acquiesced in his verdict. If Legendre's proof has been generally considered unsatisfactory, why did none of those by whom such a view was taken show where the reasoning is defective