Mathematical Department.

UNIVERSITY OF TORONTO.

MATRICULATION—1881.

ARITHMETIC AND ALGEBRA.

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1. Define the terms "abstract" and "concrete" as applied to numbers.

Is $6\times3=18$ a correct solution of the question: What will be the cost of six postage stamps at three cents each?

Book-work. Yes, if we are careful to understand 3 as an abstract and not as a concrete number. Multiplication is merely a short way of doing a peculiar kind of addition in which the addends are all equal. The multiplier is the number which shows how often this addend is repeated, and cannot therefore stand for anything but the *number of times*. It cannot, for instance, be three cents. But $6\times3=18$ may be explained thus:—If the price were one cent each, the cost would be $1\times6=6$ cents, but as the price is three times greater than one cent, the cost is 6 cents $\times3=18$ cents, where 3 is a purely abstract number.

2. Define the numerator and denominator of a fraction, and from your definition prove $\frac{3}{4}\div5=\frac{3}{20}$.

When a unit is divided into equal parts there are two things to be considered, viz: (1) The NUMBER of equal parts, (2) The SIZE of these equal parts. The numerator is the number used to express the former, and the denominator the latter of these two things.

$\frac{3}{4}\div5=$ quotient. $\therefore\ \frac{3}{4}=5$ times quotient. $\therefore\ \frac{3}{4}\times4=20$ times quotient. Now to multiply 3 fourths by 4, take the same number of parts, but increase the size of each part by 4 times; *i.e.* instead of 3 fourths take 3 units. So then we get $3=20$ times quotient, or quotient $=\frac{1}{20}$ of $3=\frac{3}{20}$ of $1=\frac{3}{20}$.

3. Prove the rule for pointing in the extraction of the cube root of a number.

There is a metal cubical box of 96 feet surface and $1\frac{1}{2}$ feet thickness; also three solid cubes of another kind of metal whose surfaces are as the numbers 1, 4 and 9, and whose combined weight equals that of the box. Find the lengths of the edges of the cubes, the weight of the latter metal being to an equal bulk of the former as 3:4.

Book-work.

Surface of one face of cubical box $=16\ \therefore$ external edge $=4$, internal edge $=\frac{5}{2}$.

Solidity of box $=4^3-(\frac{5}{2})^3=\frac{387}{8}=\frac{387}{8}\times\frac{4}{3}=\frac{129}{2}$ cub. ft. of the second kind of metal.

Now in cubes the sides are as 1.2.3, and their solidities are say x^3, $8x^3$, $27x^3$, *i.e.* their mass $=36x^3$, and $=\frac{129}{2}$ cub. ft.

$\therefore x=\sqrt[3]{\frac{129}{72}}=\frac{1}{6}\sqrt[3]{387}=1{\cdot}21456$; and the sides are

1·21456, 2·42912 and 3·64368.

4. \$500.00 is offered by a Building Society to be repaid in two annual instalments of \$285·00 each, so that the debt is liquidated at the end of two years from the present. Find the Society's rate of interest.

We have $285\{1+(1+r)\}=500(1+r)^2$

i.e. $57\,(2+r)=100(1+r)^2$

or $100r^2+143r-14=0$

$$\therefore r=\tfrac{1}{200}\{-143\pm\sqrt{(143^2+400\times14)}\}$$

$$\therefore \text{Rate }\%=\tfrac{1}{2}(-143\pm161.4)=9.2.$$

The lower sign is inapplicable to the problem.

5. A bank wishes to realize 4 per cent. interest on its discounting operations. Form a table of the rates at which it must discount notes payable in 30, 60 and 90 days respectively.

Omitting days of grace, as included in given times, and taking 360 days $=$ year, after the manner of banks, the times are $\frac{1}{12}$, $\frac{1}{6}$ and $\frac{1}{4}$ of a year respectively. And the interest of \$1 for these times at 4% is $\frac{1}{300}$, $\frac{2}{300}$, and $\frac{3}{300}$ respectively; and the amounts $\frac{301}{300}$, $\frac{302}{300}$, and $\frac{303}{300}$. Hence the P.W. in each case is $\frac{300}{301}$, $\frac{300}{302}$, and $\frac{300}{303}$ of the face of the note.

The discounts are therefore

$\frac{1}{301}$ for 30 days, $\frac{2}{302}$ for 60 days, $\frac{3}{303}$ for 90 days.

6. State the advantages arising from the employment of Bills of Exchange. Define "Par of Exchange" and "Course of Exchange"; mention any causes that influence the latter.

Book-work. In answer to the last part, we may mention a high protective tariff. This will increase the cost of all imports into the protected country, and thus increase the demand for bills and raise the rate of exchange in the protected country.

7. Prove the following:—

The difference between any common number of three digits and a number consisting of the same three digits in reversed order, is divisible by nine, eleven, and the difference of the extreme digits.

Every number of four places, in which two like significant figures have two cyphers between them is divisible by seven, eleven and thirteen.

Let a, b, c be the digits

(1). $\therefore N_1=100a+10b+c$

$N_2=100c+10b+a$

difference $=100(a\backsim c)-(a\backsim c)$

$=99(a\backsim c)$, hence the proposition.

(2). Let a be the digit at the right and left of the given number,

$\therefore$ Number $=1000a+a=1001a=7\times11\times13a$.

8. Extract the square root of

$\frac{1}{4}x-\frac{2}{3}x^{\frac{1}{2}}+\frac{25}{36}-\frac{1}{3}x^{-\frac{1}{2}}+\frac{1}{16}x^{-1}$

If this is a perfect square it consists wholly of two sorts of terms viz., square terms, like a^2, b^2 &c., and double products, like $2ab$, $2ac$ &c. The negative terms cannot be squares; $\frac{1}{2}x^{\frac{1}{2}}$ and $\frac{1}{4}x^{-\frac{1}{2}}$ are the square roots of the extreme terms, twice their product is $\frac{1}{4}$, subtracting this from $\frac{25}{36}$ we have $\frac{16}{36}$ the other square term. Having regard to the signs of the two remaining terms we see that the square root is $\frac{1}{2}x^{\frac{1}{2}}-\frac{2}{3}-\frac{1}{4}x^{-\frac{1}{2}}$.

9. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\cdots\cdots$, then

$$\frac{a^n+c^n+e^n+\cdots\cdots}{b^n+d^n+f^n+\cdots\cdots}=\frac{(a-c+e-\cdots\cdots)^n}{(b-d+f-\cdots\cdots)^n}.$$

State the general theorem of which this is a particular case:—

(1) Let $a=bx$, $c=dx$, &c.

$\therefore a-c+e$ &c. $=x(b-d+f-\ldots)$

$$\therefore x^n=\left(\frac{a-c+e-}{b-d+f-}\right)^n \quad \text{(A)}$$

Also $a^n=b^nx^n$, $c^n=d^nx^n$, &c.

$$\therefore x^n=\frac{a^n+c^n+e^n+\cdots\cdots}{b^n+d^n+f^n}=\text{(A)}$$

(2) If $\frac{a}{b}=\frac{c}{d}$, then any fraction whatever formed by combining a and b or any of their powers, is equal to a similar and similarly formed fraction from the powers of c and d.

10. Give the different methods that may be employed in the solution of simultaneous equations.

Solve $\left\{\begin{array}{l} ax+by=c \\ a_1x+b_1y=c_1 \end{array}\right\}$

Interpret your results when (1) $\frac{a}{a_1}=\frac{b}{b_1}$; (2) $\frac{a}{a_1}=\frac{b}{b_1}=\frac{c}{c_1}$

The methods in common use are (1) Method of comparison, *i.e.* finding the value of x in each equation and putting these values equal. (2) Substitution of the value of x in one equation, in the remaining equations. (3) Method of Indeterminate Multipliers. (4) Cross-Multiplication.

Multiply 2nd. equation by m and add it to 1st.

$\therefore x(a+ma_1)+y(b+mb_1)=c+mc_1$ (A)

Now give m such a value as shall cause the coefficient of one of the unknowns to vanish, *e.g.* put $a+ma_1=0$, i.e. $m=-\frac{a}{a_1}$. Substitute this value of m in (A) and we get $y=\frac{a_1c_1-ac_1}{a_1b-ab_1}$. Similarly by putting $b+mb_1=0$, we get $x=\frac{b_1c-bc_1}{a_1b-ab_1}$

(1) If $\frac{a}{a_1}=\frac{b}{b_1}$ then $a_1b-ab_1=0$ and $x=y=\infty$.