2. *Why is such knowledge considered as a requisite for general cultivation?*

Because the whole mass of bodies, the universe, as well as man, exists in space; because without the knowledge of the qualities of space, man would be ignorant of that appearance of things which belong to their inmost nature; because geometry teaches how to measure lines, surfaces and bodies, which knowledge is very necessary; because without it man could not divine, that the distance and size of the sun, moon and stars, could be determined; and because he would even have no idea of the extent of his own abode, and of the mathematical, i. e., fundamental qualities of the same. All this is consequently requisite for general human cultivation, not to speak of its practical value, as well for female as male education, and therefore for the common school, the school of the people. Without it, not the most indispensable part, but an essential part, of education is wanting.

3. *What elements of geometry are to be taught in the common school? and in general what parts of it may be considered there?*

Space admits of "intuitive," (*anschauliche,*) and a demonstrative, (*begriffsmaessige,*) observation.

The intuitive faculty of man perceives immediately objects in space, bodies in their qualities and forms; with the sense of touch he perceives what opposes him in space, the body and its external form; the sense of sight assists him, by determining extent and distance, and by comparing and measuring them. These are operations of *external* intuition. The intellect abstracts the *differentia* of the bodies, and fixes the pure, mathematical form; and thus aids the *interior* pure, or mathematical intuition. Moreover, the logical intellect, perceiving the dependence of magnitudes on each other, their mutual conditions, the inference of the one from the other, deduces and concludes.

The intuitive part of geometry is that elementary part which is proper for the common school. But thereby is not meant, that the pupils should not learn the dependence of one thing on the other; this even can not be avoided, it comes of itself; but according to the degree of ability, quicker and deeper with one than with another, and one school will make more progress in it than another. But the power to be immediately employed is the faculty of observing—first, the exterior, and then, and preëminently, the interior. The conclusions connected with that observation result therefrom spontaneously; the intellect works without being ordered. Therefore, in geometry, as every where—a fact, ignorance of which, causes much merely repetitions and lifeless teaching, as well as intellectual dependence and immaturity—the teacher ought to lead the scholar to immediate, true and vivid perceptions.

The strict or Euclidean geometry, with its artificial proofs, is not fit for the common school, nor does it prosper there.

4. *What is more particularly the subject of geometrical instruction in the peoples' school?*

The qualities of (mathematical) lines, surfaces and solids.

5. *What method is to be pursued with it?*

The point of starting is taken in the physical body; and from this the mathematical one is as it were distilled.

The order of single precepts or propositions is, as has been said, as much as possible *genetical*. Pedantry and anxiety are here, as every where, prejudicial. The method, always intuitive, requires originality, i. e., the evolving of every thing learned from some thing preceding; aims at immediate spontaneous understanding of one thing *through* the other.

6. *What is the immediate purpose of this instruction?*

To understand the qualities of lines, plains and bodies; to measure and calculate them.

7. *What instruments are used by the pupil?*

Pen and pencil, for drawing; compass and scales, for measuring; the usual measures of lines, surfaces and bodies, for calculating.—(*Barnard's, American Journal of Education.*)

(*To be continued.*)

Lessons in arithmetic.

ON VULGAR FRACTIONS.—No. 2. (1.)

V. Addition and subtraction of fractions:—Addition has been defined as the process of finding *one number,* called the *sum,* which shall be exactly equal to two or more numbers. From this definition it follows that in order to add numbers representing objects, they must be of the same *kind or denomination;* for example it is evident we cannot express in one sum 3 *oranges* x 5 apples; before the addition can be effected, but no difficulty exists in the following cases 3s. x 5s., = 8s., 10 *marbles* x 15 *marbles* = 25 *marbles*.

Now let us endeavour to apply the above in the case of the addition of two fractions. Suppose an apple to be divided into 9 equal parts, 3 of these parts will be 3-*ninths* and 5 parts, 5-*ninths*, and it is plain that the sum of 5-*ninths*, and 3-ninths is 8-*ninths*; for as in each case unity is divided into the same number of parts, each part is of the same size or value, and we wish to find the sum of 3 and 5 of those parts. Therefore when the denominators are alike, we simply add the numerators and retain the same denominator for our new fraction. Again, let it be required to find the sum of 2/3 and 3/4. Here the denominator of each fraction is different, and consequently the size or value of the part is different; that is, 2 parts of unity of a certain size are to be added to 3 parts of a different size. Therefore, 2-*thirds* and 3-*fourths* cannot be added while the fractions remain respectively *thirds and fourths*, any more than £2 and 3 crowns can be added, so long as the £'s remain pounds, and the crowns remain crowns. In the latter case however the addition can be effected by expressing the value of the pounds and crowns by an equivalent number of some *common* coin, as the shilling, of which the pound and crown are both multiples. In like manner 2/3 can only be added together, when they are expressed as fractions, whose denominations are some *common part of unity* of which *one-third* and *one-fourth* are respectively multiples. Now we have shown that any fraction may be expressed in a variety of forms by multiplying the numerator and denominator by the same number, and it is easy to select two numbers one of which multiplied into the numerator and denominator of the first fraction and the other into the numerator and denominator of the second, shall reduce the fractions to a *common denominator.*

Thus the L. C. M. of 3 and 4 = 12
and 2/3 = 2x/3x1 = 8/12 also 3/4 = 3x8/4x1 = 9/12
therefore 2/3 x 3/4 8x9/12 = 17/12

Or it may be demonstrated by taking a line and dividing it, that 1/3 = 4/12 and therefore 2/3 = 8/12; also, that 1/4 = 3/12 and therefore 3/4 = 9/12 Hence to add fractions, reduce them to a common denominator add the numerators, and retain the common denominator for the new fraction. In the same manner it can be proved that to subtract one fraction from another they must be reduced to a common denominator. For it is evident we cannot compare quantities referring to different things. Hence the rule will be similar to that for addition, viz,—Reduce the fractions to a common denominator, subtract the numerators, and retain the common denominator.

VI. *Multiplication of fractions.*

We have already considered the case of the multiplication of a fraction by a whole number, and it now remains to consider the general case of the multiplication of a fraction by a fraction. Take, for example, the two fractions 3/4 and 5/7, and let it be required to find the product of 5/7 multiplied by 3/4. Now multiplication is defined as the addition of a number to itself as many times as is indicated by the multiplier: thus, 3 times 4 means 4 added to itself 3 times, as it is impossible to add 5/7 to itself 3/4 times or 3/4 of a time. To ascertain then, what is meant by 5/7 multiplied by 3/4, we must understand exactly what 3/4 means as that is our multiplier. Now we have shown that every fraction has two meanings, and according to the second of these 3/4 equal 1/4 of 3; therefore 3/4 x 5/7 is the same thing as 1/4 of 3 x 5/7. But 3 x 5/7 is 5/7 added to itself three times or 15/7, and 5/7 multiplied by 3/4 or 1/4 of 3 must give 1/4 of this result, which will evidently be the required product, viz., 15/28. It will be observed that this result has really been obtained by multiplying the numerators and denominators of the fractions together. And it will be seen, that, to multiply any quantity by a fraction, is to add that quantity to itself, as many times as there are units in the numerator of the fraction, and to take such a part of this result, as is indicated by the denominator of the fraction.

VII.—*Division of Fractions.*—To divide one quantity by another, is to find how many times the latter is contained in the former. It is evident, from this definition, that numbers can only be compared by this rule when they refer to objects of the same kind. A number of *days*, for instance, cannot be contained any number of times in a number of *acres;* but 3s is contained in 15s. five times, &c. To divide then 8/9 by 4/9 is to find how often 4 *ninths* is contained in 8 *ninths*. The answer is evidently 2 *times* or *twice;* for 8/9 are exactly twice as many ninths as 4/9. Again let it be required to divide 3/4 by 4/5. Reducing the fractions to a common denominator, 3/4 divided by 4/5 is the same thing as 15/20 divided by 16/20, and from the preceding 16/20 is contained in 15/20 as often as 16 is contained in 15 i. e. the quotient obtained by the division of 15/20 is the same as the quotient obtained by the division of 15 by 16. Now we have proved that a fraction expresses the quotient obtained by

(1) See our issues for October and December 1857.