From curve A the deflection of the arch (Fig. 3) at the ⅓ point can be directly ascertained. It is found to be 0·132 in. If the cantilever 250 feet high and 1 foot wide were actually forced to deflect 0·132 in. at this point (Elev. 166·67 feet) a force, F, would be required which can be found as follows (F is concentrated at the ⅓ point):

$$D_o = \frac{n F l^3}{E h^3} \quad - \quad - \quad - \quad - \quad (8)$$

In (8) (taken from standard handbooks) the value of n depends upon the rate of variation of the face slopes. If both faces were vertical n would equal 4. If the faces (or at least the downstream face) were shaped as flat parabolas, or if the thickness of the section in an upstream and downstream direction at the one-third point was approximately half the thickness at the foundation, n would equal 8.

This last condition is the one that theoretically best fits cases in dam construction. Considerable modifications are mostly necessary, however, due to the fact that the rock foundation itself, to some extent, takes part in the movements of the dam body. With a full water load the rock foundation under the middle portion of an arch dam moves more in a downstream direction than does the ends, as the push in a downstream direction is the greatest in the middle and as at the ends, only a component of the axial compression acts in a downstream direction. Therefore, the cantilever can not take up as great a proportion of the water load as it would if fastened to an immovable foundation and more load is therefore thrown on the arch. The writer has for some time been trying to find a practical value for n by analyzing deflection data obtained from actual dams. He thinks he is justified in using $n = 12$ for solid rock foundation, and 16 for seamy rock foundation. This makes (8) empirical, but the results from it are believed to be closer to actual facts than any results arrived from mere theoretical conditions on account of the number of assumptions it is necessary to make.

Inserting the value of 12 for n in (8):

$$D_o = \frac{0{\cdot}132}{12} = \frac{12 \times F \times 83{\cdot}33^3}{432{,}000{,}000 \times 110^3}$$

$$F = 911{,}000 \text{ lbs.}$$

The cantilever will deflect the same as the arch when thus loaded.

The total water load on a vertical slice of the dam, 1 foot wide and 250 feet high, is $250 \dfrac{0{\cdot}15{,}625}{2} = 1{,}953{,}125$ lb. The initial stress supports $1{,}953{,}125 \times \dfrac{16{\cdot}4}{100} =$ 320,312 lb. before any deflection takes place. Therefore the load causing a deflection of 0·132 in. of the combined arch and cantilever must be equal to 911,000 + 1,953,125 — 320,312 = 2,543,813 lb. The proportion of this amount taken by the cantilever will be $\dfrac{911.000}{2{,}543{,}813} = 35{\cdot}8$ per cent.

Now, the actual load to be divided between cantilever and arch is not 2,543,813 lb. per running foot, but only 1,953,123 — 320,312 = 1,632,813 lb. Of this amount the cantilever carries 35·8 per cent., or $1{,}632{,}813 \times \dfrac{35{\cdot}8}{100} =$ 584,547 lb., concentrated at the one-third point, making the actual deflection at this point $0{\cdot}132 \times \dfrac{64{\cdot}2}{100} = 0{\cdot}0847$ in.

The bending moment due to this force is equal to 584,547 × 83·33 = 48,710,301 ft.-lb.

The section modulus of the base $= \dfrac{110^2}{6} = 2{,}011$, and therefore the compressive stress on the foundation at the toe, due to the bending action of the water load on the cantilever, is equal to

$$\frac{\text{Bending moment}}{\text{Section modulus}} = \frac{48{,}710{,}301}{2{,}011} = 24{,}222 \text{ lb. per sq. ft.} \quad (9)$$

The total compression on the foundation at the toe will be this compression added to that due to the weight of the structure, which amount to approximately 16,200 lb. per sq. ft. at the toe, making the total compression approximately 40,400 lb. per sq. ft.

If a base length of 70 ft. is chosen, the arch would take a greater percentage of the load and the curved beam a smaller, leaving the same or less for the cantilever, but, owing to the smaller section modulus of the 70-ft. base, the compression at the toe would be somewhat higher than 24,222 lb. per sq. ft., and the compression due to the weight of the structure would be much higher than

$$CC_c = \frac{\frac{2\sin\theta}{\theta}(1-\cos\theta)+\frac{1}{2}(\cos 2\theta-1)}{\frac{3\theta}{\sin\theta}+\cos\theta-4}+(1-\cos\theta)$$

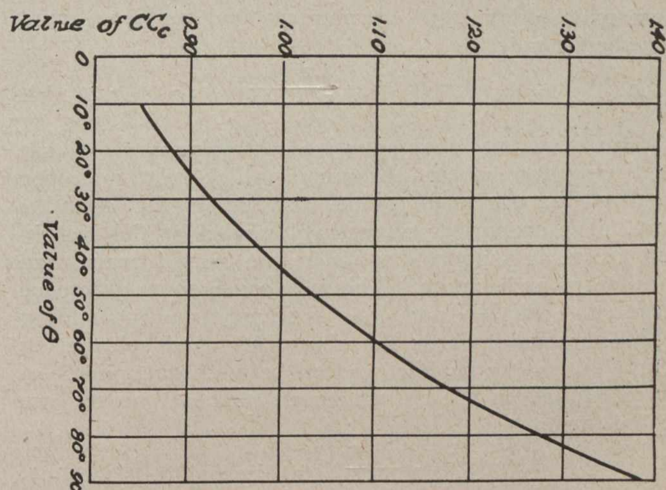

Fig. 5.

16,200 lb. per sq. ft., so that the sum of the two would be considerably more than 40,400 lb. per sq. ft. Although within the safe limit, the resulting vertical compression would be somewhat out of proportion to the 36,000 lb. per sq. ft. (and less) axial compression used when calculating t from (1).

The dam section with the 110-ft. base contains only 4% more material than the dam with the 70-ft. base (Fig. 3), as the addition is not made as a portion of a circular ring, but in the shape of a spherical triangle. Any intermediate base length between the two limits given in Fig. 3 could be accepted for a dam built on this particular site. The two stresses (the 36,000 lb. per sq. ft. average axial compression, and the maximum 40,400 lb. per sq. ft. vertical compression) are acting in planes perpendicular to each other, and therefore tend to support each other. Although they are low, the resulting section (Fig. 3) appears slender on account of the economical distribution of the material.

This method of calculating the vertical stress upon the foundation is correct only so long as no tension exists at the heel, or if tension exists, as long as this tension is properly taken care of. For the constant angle arch,