

the measurement would produce the estimate that $N_E = 230$. For hypothesis A, Prior Probability x Likelihood =

$$0.6 \times 0.000443 = 0.000266.$$

When the calculations are repeated for hypotheses B and C, it is seen that the probability of estimating $N_E = 230$ is nearly three times higher under hypothesis B than A, and is negligible under hypothesis C. Since measurement has resulted in the estimate 230, the probability that N_E is 230 is now 1. This is "normalized" by multiplying the separate probabilities that A is true, that B is true, and that C is true by a common factor, raising the total probability to 1. The "posterior probability" $\Pr(B | N_E = 230)$ that hypothesis B is true is therefore estimated to be 0.73, with the probabilities for A being:

$$\Pr(A | N_E = 230) = 0.27,$$

and for C, $\Pr(C | N_E = 230) = 0.00.$

The calculated example above was for the particular case in which the observations produced the estimate $N_E = 230$. Figure 9 plots these probabilities for values of N_E , ranging from below 180 to beyond 320.

The three distribution curves at the top of Figure 9 show the probabilities of estimating the indicated value of N_E , assuming the prior probabilities for the three hypotheses A, B, and C. They are bell-shaped curves (of the normal distribution) centred on the estimated numbers 200, 250 and 300, and with areas proportional to the prior probabilities 0.6, 0.1, and 0.3. It can be seen that only hypothesis A is at all likely to produce an estimate N_E less than 220, only B an estimate between 235 and 260, and only C one above 280. But either hypothesis A or B could produce a value of N_E between 220 and 235, and either B or C for $260 < N_E < 280$.

The three distribution curves at the bottom of Figure 9 show the posterior probabilities that hypothesis A, B or C is true, after using the information that the measurement has produced its estimate N_E . For example, the curve labelled $\Pr(A | N_E)$ indicates that for $N_E < 215$, the probability that hypothesis A is true is virtually a certainty; but in the range $215 < N_E < 240$, it drops to nearly 0. At $N_E = 228$, the posterior probability that hypothesis B is true has exceeded that for A. But when $N_E = 272$, it is more probable that C, rather than B, is true.

It can be seen that if $0 < N_E < 225$, one can infer that $N = 200$; if $230 < N_E < 265$, N is probably 250 (almost certainly 250 if $240 < N_E < 260$); while if $N_E > 275$, then N can be inferred to be 300. If N_E is in the range 225-230, it is not possible to choose decisively between hypotheses A and B, while if $265 < N_E < 275$, B or C may be true.

This provides a simple example of synergy between observed (objective) and other (possibly subjective) factors.

