the primitive 2^d root of unity. According to our usual notation, let P_z, ϕ_z, etc., be what P_1, ϕ_1, etc., become when w is changed into w^z, z being any integer. Then, from (104),

$$\left.\begin{aligned} R_z &= A_z^n (P_{zm}^m \phi_{z\sigma}^\sigma \psi_{z\tau}^\tau \ldots\ldots F_{z\beta}^\beta) \\ R_z^{\frac{1}{n}} &= w' A_z (P_{zm}^m \phi_{z\sigma}^\sigma \psi_{z\tau}^\tau \ldots\ldots F_{z\beta}^\beta)^{\frac{1}{n}} \end{aligned}\right\} \tag{108}$$

Therefore

w' being an n^{th} root of unity. The general primitive n^{th} root of unity being w^e, give w' in the second of equations (108) the value unity for every value of z included under e. Then

$$R_e^{\frac{1}{n}} = A_e (P_{em}^m \phi_{e\sigma}^\sigma \psi_{e\tau}^\tau \ldots\ldots F_{e\beta}^\beta)^{\frac{1}{n}}. \tag{109}$$

Taking any number y distinct from n in the series (107), since y is a factor of n, let $yv = n$. Then w^v is a primitive y^{th} root of unity. Hence, since w^e is the general primitive n^{th} root of unity, all the primitive y^{th} roots of unity are included in w^{ev}. If w' in the second of equations (108) be w^a when $z = v$, give w' the value w^{ea} when $z = ev$. Then

$$R_{ev}^{\frac{1}{n}} = w^{ea} A_{ev} (P_{evm}^m \phi_{ev\sigma}^\sigma \ldots\ldots F_{ev\beta}^\beta)^{\frac{1}{n}}. \tag{110}$$

The expression P_m having the form of the fundamental element of the root of a pure uni-serial Abelian quartic, it is understood that, in (110), $P_{evm}^{\frac{m}{n}}$ or $P_{evm}^{\frac{1}{4}}$ is taken with the value which it has in the root

$$P_0^{\frac{1}{4}} + P_m^{\frac{1}{4}} + P_{2m}^{\frac{1}{4}} + P_{3m}^{\frac{1}{4}}$$

of a pure uni-serial Abelian quartic; and consequently, when v is a multiple of 2, w^{ma} must have the value unity. Form equations similar to (110) for the remaining terms in (107). In this way, because the series of the n^{th} roots of unity distinct from unity is made up of the primitive n^{th} roots of unity, and the primitive y^{th} roots of unity, and so on, all the terms $1, 2, \ldots\ldots, n-1$ will be found in the groups of numbers represented by the subscripts e, ev, etc., when multiples of n are rejected. Consequently, in determining $R_e^{\frac{1}{n}}$, $R_{ev}^{\frac{1}{n}}$, etc., as in (109), (110), etc., we have determined all the terms

$$R_1^{\frac{1}{n}}, R_2^{\frac{1}{n}}, \ldots\ldots, R_{n-1}^{\frac{1}{n}}. \tag{111}$$

Substitute, then, in (105) the rational value of $R_0^{\frac{1}{n}}$, and the terms in (111) as these are determined by the equations (109), (110), etc., and the root is constructed; that is, the expression (105) is the root of a pure uni-serial Abelian equation of the n^{th} degree, provided always that the equation of the n^{th} degree, of which it is the root, is irreducible.