

§31. In the article of the *American Journal of Mathematics* (Vol. VI, page 103) referred to in the opening paragraph of this paper, the solvable irreducible quintic in which u_1u_4 is equal to u_2u_3 was discussed, and the roots of the equation were shown to be determinable in terms of the coefficients p_2, p_3 , etc., even while these coefficients have no definite numerical values assigned to them, but remain symbolical. The solution that has now been given is much simpler than the former; equally with the former, it is applicable to equations with symbolical coefficients, the assumption being of course made that the coefficients are related as in (49); and it possesses the advantage of being part of a general theory.

ADDITIONAL EXAMPLES.

§32. *Sixth Example.*—Let

$$x^5 + 320x^3 - 1000x + 4288 = 0.$$

Here $g = 0, k = -16$. Because $g = 0$, we use the formulæ (11) and (16). The commensurable values of y and t which satisfy (11) and (16) are

$$y = 8, t = 6.$$

Also, by (14),

$$A = 56.$$

Therefore, from the second of equations (9) and from (20),

$$\begin{aligned} B &= -16 \times 37, B'\sqrt{z} = -16 \times 20, \\ \therefore B + B'\sqrt{z} &= -16(37 + 20\sqrt{2}). \end{aligned}$$

And $u_1u_4 = -2\sqrt{2}$. Therefore

$$\begin{aligned} x &= u_1 + u_4 + u_2 + u_3 \\ &= [-16(37 + 20\sqrt{2}) + \sqrt{\{256(37 + 20\sqrt{2})^3 - (-2\sqrt{2})^5\}}]^{\frac{1}{4}} \\ &\quad + [-16(37 + 20\sqrt{2}) - \sqrt{\{256(37 + 20\sqrt{2})^3 - (-2\sqrt{2})^5\}}]^{\frac{1}{4}} \\ &\quad + [-16(37 - 20\sqrt{2}) + \sqrt{\{256(37 - 20\sqrt{2})^3 - (2\sqrt{2})^5\}}]^{\frac{1}{4}} \\ &\quad + [-16(37 - 20\sqrt{2}) - \sqrt{\{256(37 - 20\sqrt{2})^3 - (2\sqrt{2})^5\}}]^{\frac{1}{4}}. \end{aligned}$$

§33. *Seventh Example.*—Let

$$\left(\frac{x}{10}\right)^5 + 40\left(\frac{x}{10}\right)^3 - 69\left(\frac{x}{10}\right) + 108 = 0,$$

or,

$$x^5 + 40000x^3 - 690000x + 10800000 = 0.$$

Here $g = 0, k = -2000$. Because $g = 0$, we use the formulæ (11) and (16).