sq. root, (see Arith. p. 73.) But the subtrahend immediately producing a remainder $= ab + b^2$, where b is the no. represented by the digit last obtained (thus far) in the root and a, those previously obtained.

91. Between the sq. of the part of the root already found and the no. whose sq. root is to be obtained.

92. Between the cube of the part of the root already found and the no. whose cube root is to be obtained. See Arith., p. 77.

93. Disregarding the dec. pt., the first 6 digits in the root are 331662, *i.e.* the part of the root already found is 33166200000 ($=$ a suppose); and if we denote the no. whose rt. is required (11 and 20 0's) by N, the next complete rem. is $N - a^2$ ($=$ 317756 and 10 0's), and the next trial divisor is 2 a ($=$ 66332400000). Now if x denote the rest of the root ($=$ 47903), $\therefore N = (a + x)^2$, $\therefore N - a^2 = 2\,a\,x + x^2$, and we are required to show that the rest of the root (x) may be obtained by dividing $2\,a\,x + x^2$ by 2 a instead of continuing the ordinary process. The quot. so obtained is $x + \frac{x^2}{2a}$ which gives the remaining part of the root x provided $\frac{x^2}{2a}$ is a proper fraction. Now since x contains 5 digits and a, 11, $\therefore x^2$ must always be less than a, $\therefore \frac{x^2}{2a}$ is less than $\frac{1}{2}$. In this division by 2 a the contracted method may of course be used and the whole operation, retaining remainders only, is as follows:—(The 30974 under the divisor is the quotient in reversed order. See Arith., p. 70.)

| | |
|---|---|
| | 11 (3.3166247903 |
| 63 | 200 |
| 661 | 1100 |
| 6626 | 43900 |
| 66326 | 414400 |
| 663322 | 1644400 |
| 663324 | 3177560 |
| 30974 | 524264 |
| | 59940 |
| | 243 |
| | 45 |